

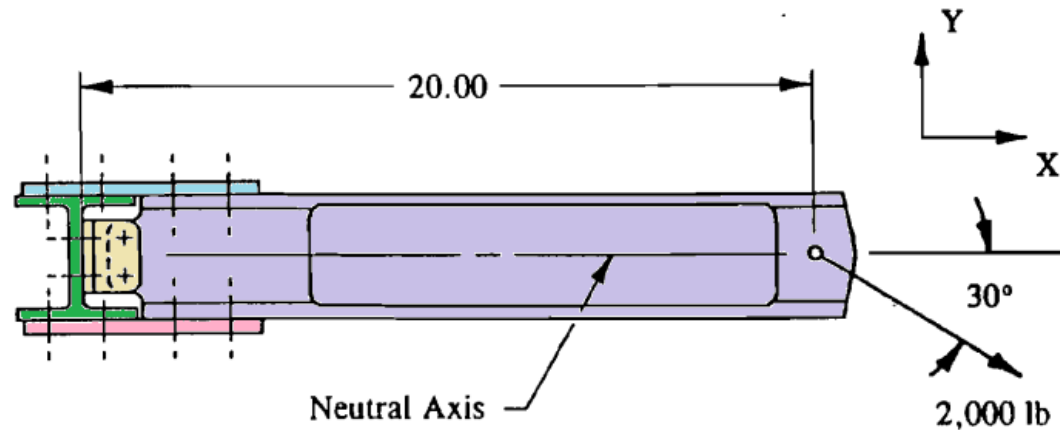


# AEE 461 Design of Aircraft Structures

Lecturer: Murat ÇELİK

## Lecture #3 Principal of Statics – cont'd

This method basically gives us internal loads at any section of a beam.

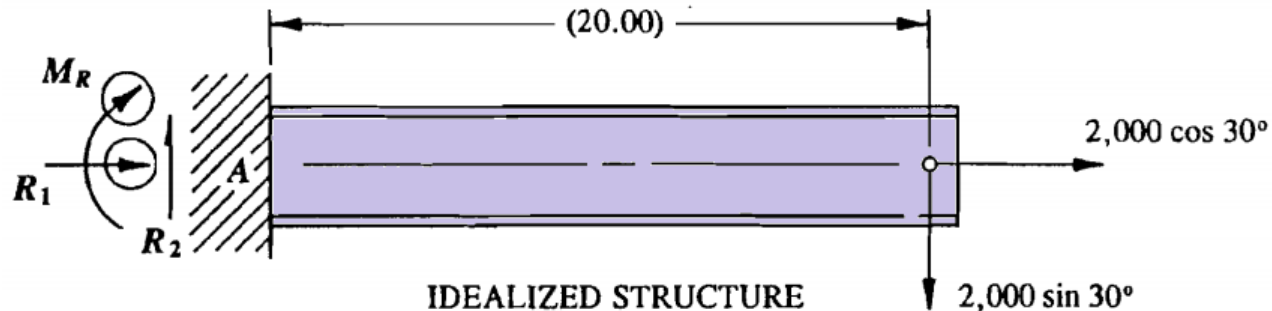


**FIGURE 1-21** An actual fixed-supported beam structure.

- The internal force  $P$  acting parallel to the axis of the beam is called the axial force.
- The internal force  $V$  acting perpendicular to the axis of the beam is called the shear force.
- The internal force  $M$  acting in the plane of the beam is called the bending moment.

Note that, internal stresses at a section of a beam that actually determines the ultimate strength of the member --not the internal forces.

The structure is «*statically determinate*», it means that three unknown reaction forces existing at this support, and three independent equations of equilibrium available for solution. So reaction forces can be calculated as in below:

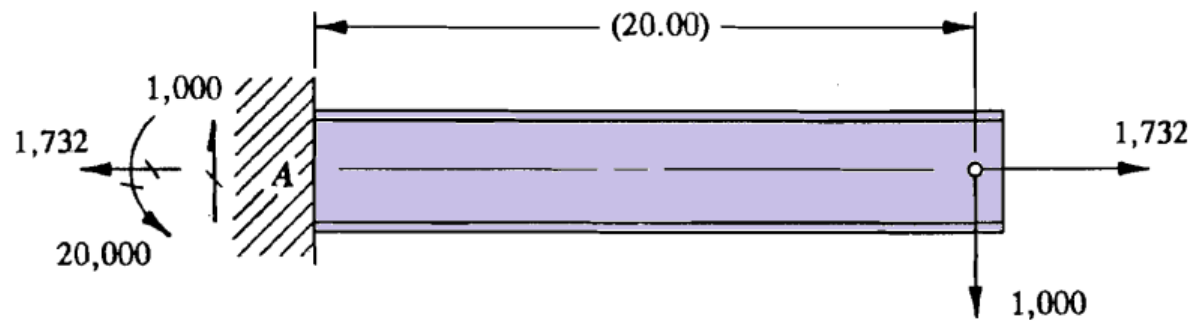


**FIGURE 1-22** Idealization of a fixed support structure using unknown reaction forces.

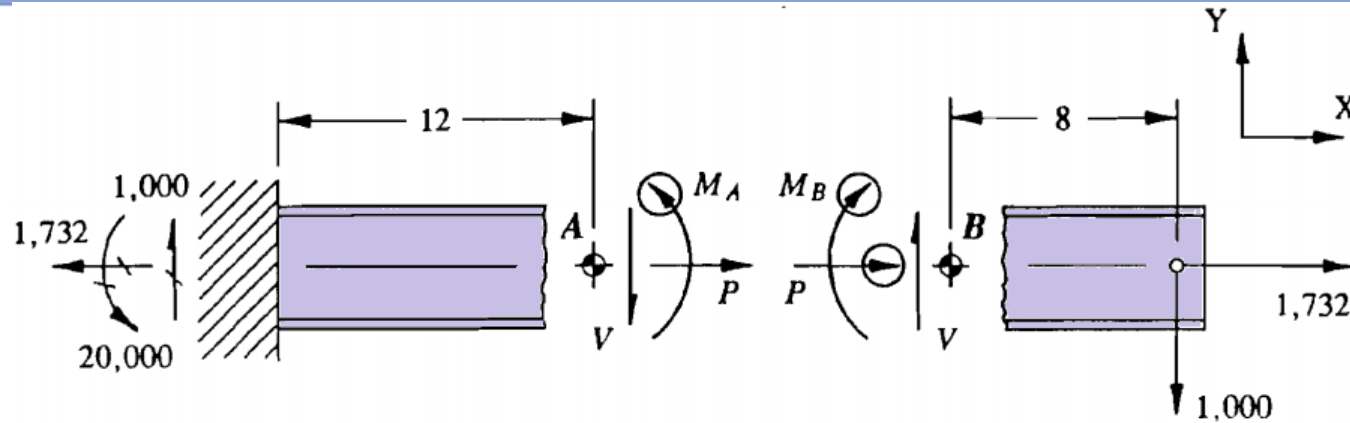
$$\Sigma F_x = 0, \quad R_1 + 2,000 \cos 30^\circ = 0, \quad R_1 = -1,732 \text{ lb}$$

$$\Sigma F_y = 0, \quad R_2 - 2,000 \sin 30^\circ = 0, \quad R_2 = 1,000 \text{ lb}$$

$$\Sigma M_A = 0, \quad \curvearrowleft M_R + 2,000 \sin 30^\circ (20) = 0, \quad M_R = -20,000 \text{ in-lb.}$$



**FIGURE 1-23** Magnitude and direction of the reaction forces determined.



**FIGURE 1-24** System of internal forces shown on exposed sections of a beam.

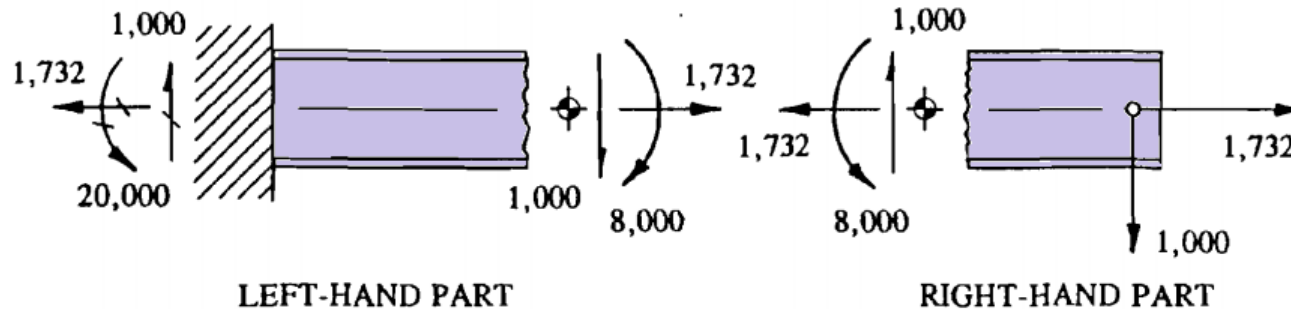
In order to show the method of sections approach, let us consider a section cut eight(8) inches from the free end of the beam. Keep in mind that, each of the isolated parts must be in static balance. So, the internal loads can be found as

Equilibrium equations for the left-hand part of the structure:

$$\begin{aligned}\Sigma F_x &= 0, & -1,732 + P &= 0, & P &= 1,732 \text{ lb} \\ \Sigma F_y &= 0, & 1,000 - V &= 0, & V &= 1,000 \text{ lb} \\ \Sigma M_A &= 0, & \curvearrowright 1,000(12) - 20,000 - M_A &= 0, & M_A &= -8,000 \text{ in-lb.}\end{aligned}$$

Equilibrium equations for the right-hand part of the structure:

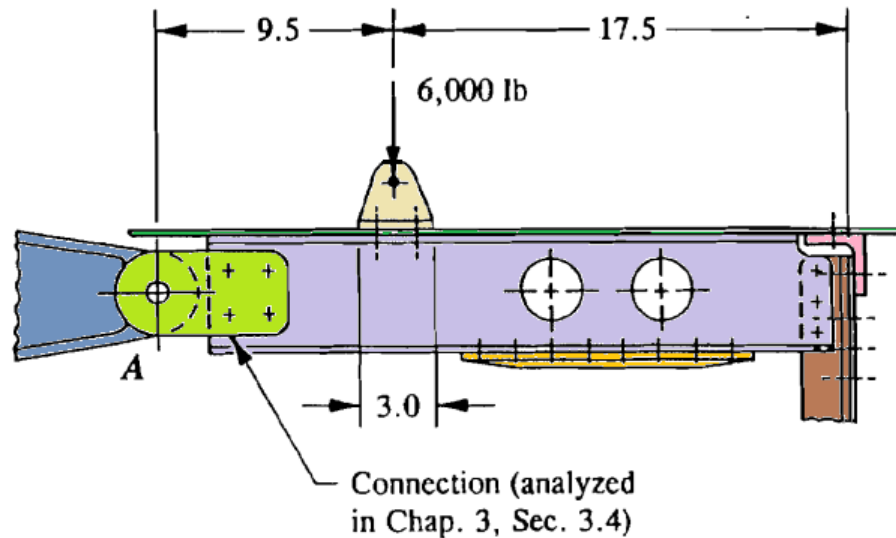
$$\begin{aligned}\Sigma F_x &= 0, & P + 1,732 &= 0, & P &= -1,732 \text{ lb} \\ \Sigma F_y &= 0, & V - 1,000 &= 0, & V &= 1,000 \text{ lb} \\ \Sigma M_B &= 0, & \curvearrowright 1,000(8) + M_B &= 0, & M_B &= -8,000 \text{ in-lb.}\end{aligned}$$



**FIGURE 1-25** Magnitude and direction of the internal forces at exposed sections of a beam.

In summary, to one of the exposed parts, three unknown internal forces are labeled without concern to their correct signs. The unknown forces are easily calculated by successfully applying the equations of statics to the isolated part. Additional section cuts are usually taken along a beam in order to understand the general loading behavior of its internal structure.

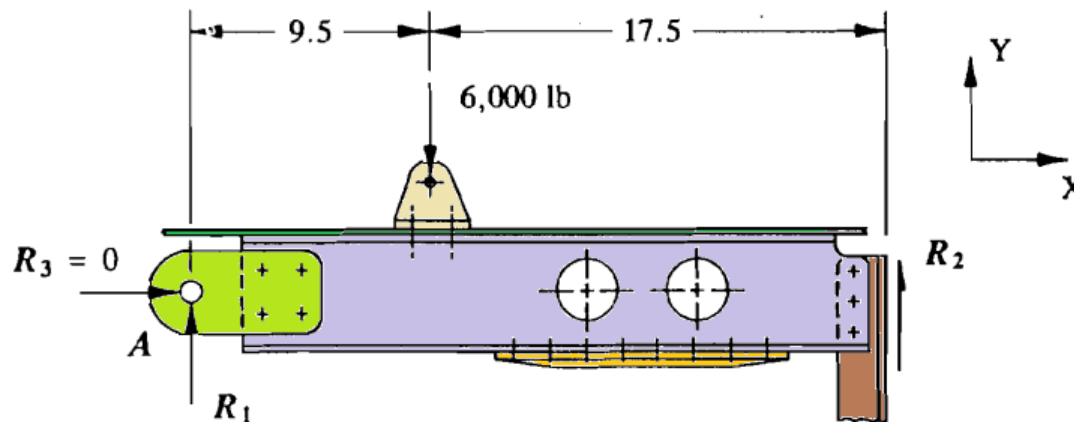
## Example 1-3:



**FIGURE 1-26** Beam structure loaded by a concentrated load through its local structure.

Determine the beam internal forces at a point 9.5 in from support A when the upper attached structure reaches its maximum design load of 6000 lb.

Solution:

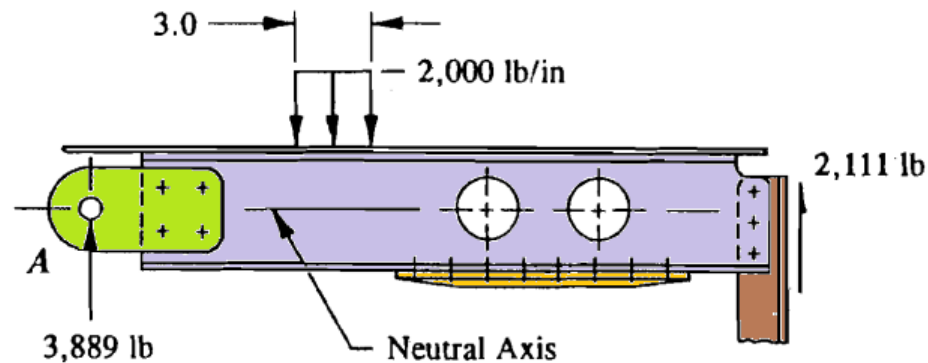


**FIGURE 1-28** Unknown beam reactions indicated on a properly drawn diagram of the beam.

Since the attached structure is fastened to the main structure over such a short span and connected via one-row rivetline, it is doubtful that this member will offer any additional stiffness or bending resistance to the overall main structure.

In order to get more realistic stresses and loads, the attached structure at LHS must have enough stiffness to transfer the lug to the main structure. Otherwise, no matter what analytical results are obtained by rigorous analysis, it will not be meaningful physically.

Solution:



**FIGURE 1-29** Balanced structure showing actual bearing distribution of the local structure.

$$\Sigma F_x = 0, \quad R_3 = 0$$

$$\Sigma F_y = 0, \quad R_1 - 6,000 + R_2 = 0$$

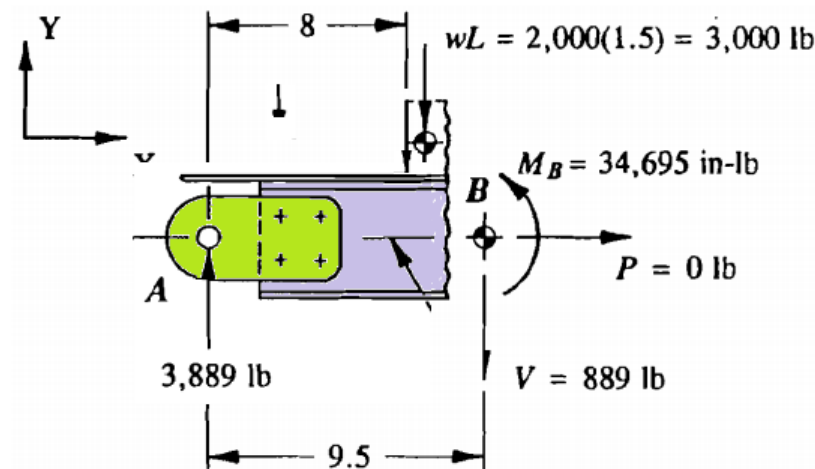
$$\Sigma M_A = 0, \quad \textcircled{+} \quad 6,000(9.5) - 27R_2 = 0, \quad R_2 = 2,111 \text{ lb.}$$

And then substituting back into the second equation gives:

$$R_1 - 6,000 + 2,111 = 0, \quad R_1 = 3,889 \text{ lb.}$$



Solution:



**FIGURE 1-30** Internal loads at an exposed section of a beam.

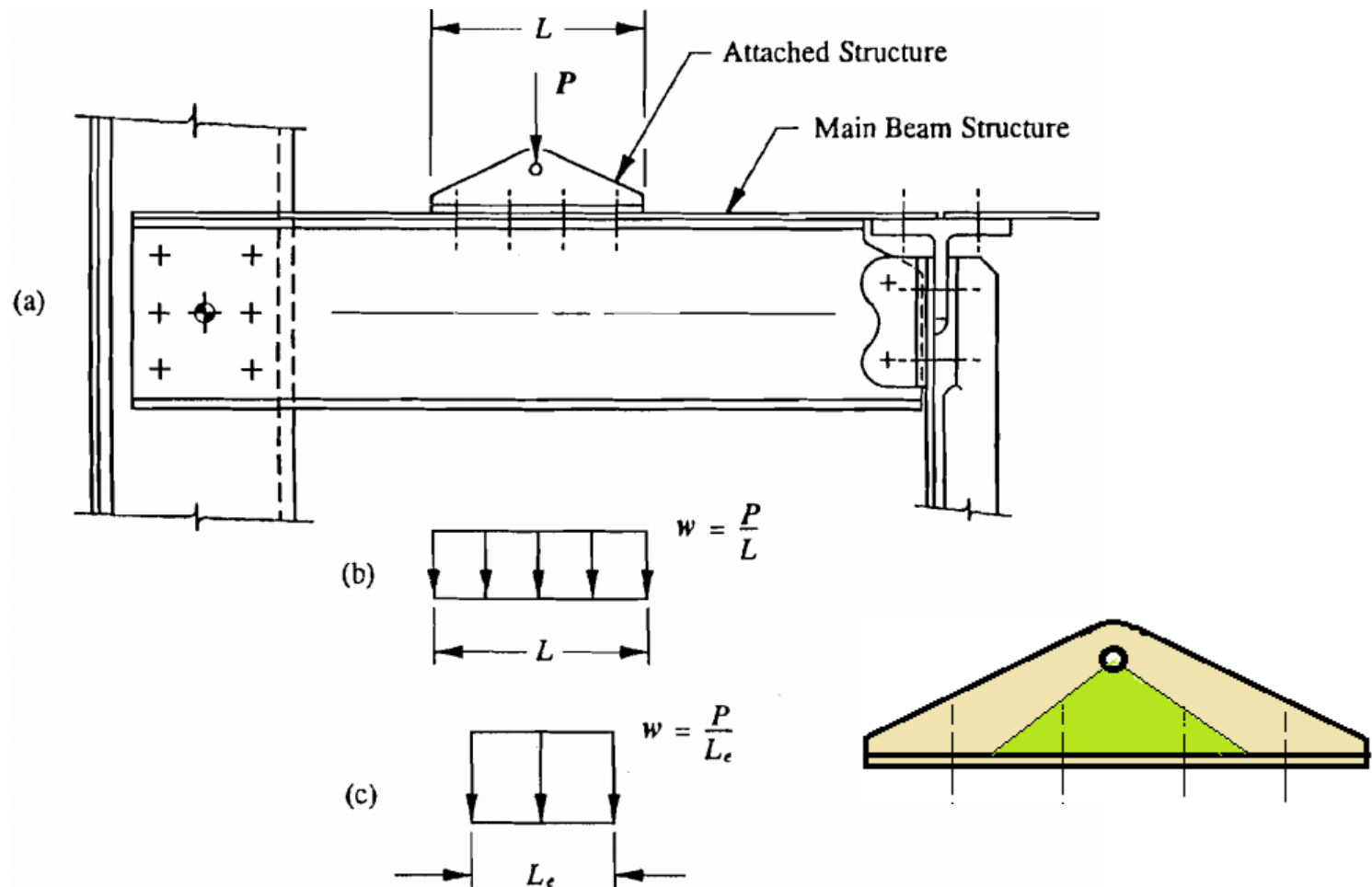
$$\Sigma F_x = 0, \quad P = 0$$

$$\Sigma F_y = 0, \quad 3,889 - 3,000 - V = 0, \quad V = 889 \text{ lb}$$

$$\Sigma M_B = 0, \quad \left( \begin{array}{c} \curvearrowright \\ + \end{array} \right) 3,889(9.5) - 3,000 \left[ \frac{1.5}{2} \right] - M_B = 0$$

$$M_B = 34,695 \text{ in-lb.}$$

## Effective length of bearing distribution for an attached structure



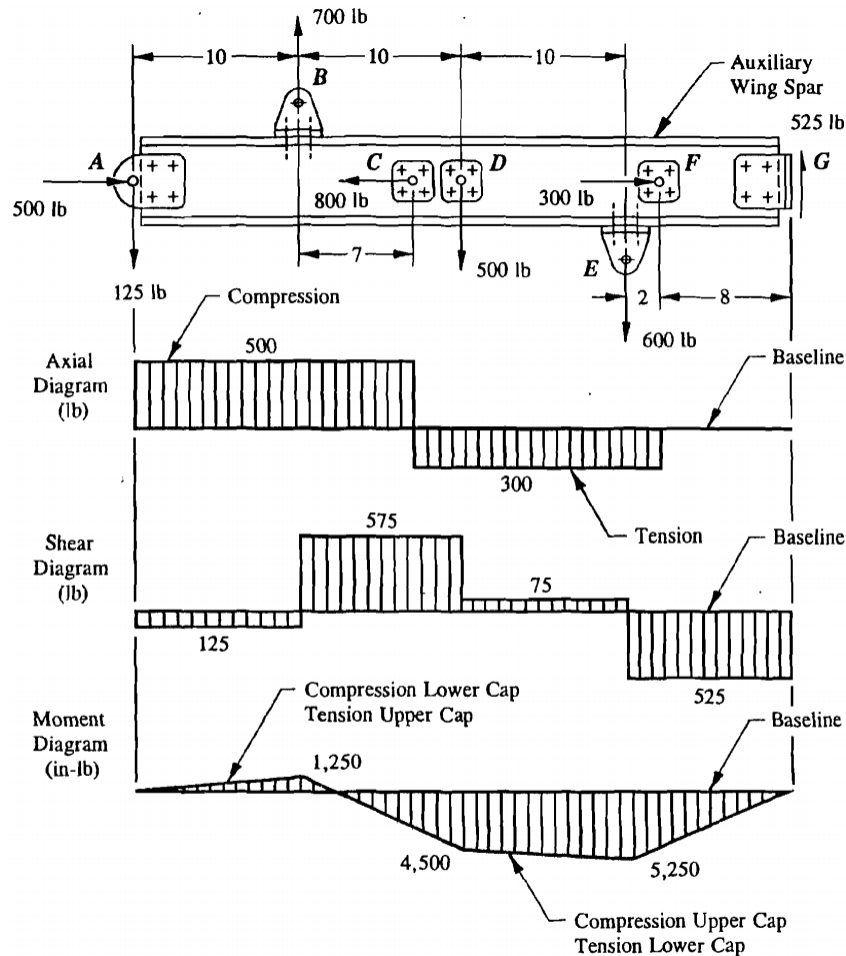
**FIGURE 1-31** Effective length of bearing distribution for an attached structure.

The magnitude of the internal forces may easily be computed at any section desired using the «**method of sections approach**». After some points have been calculated along the entire length of the beam, the internal forces are plotted for each particular kind of load they represent. These plots are called,

- Axial diagram,
- Shear diagram,
- Moment diagram

Most engineering books specify a particular sign convention to describe the internal forces of these diagrams. This usually makes a discussion of internal force diagrams only more awkward and confusing. Therefore less practical to use. Instead, we will concentrate on the actual physical effect each internal load produces on the structure being analyzed.

# AXIAL, SHEAR AND BENDING MOMENT DIAGRAMS



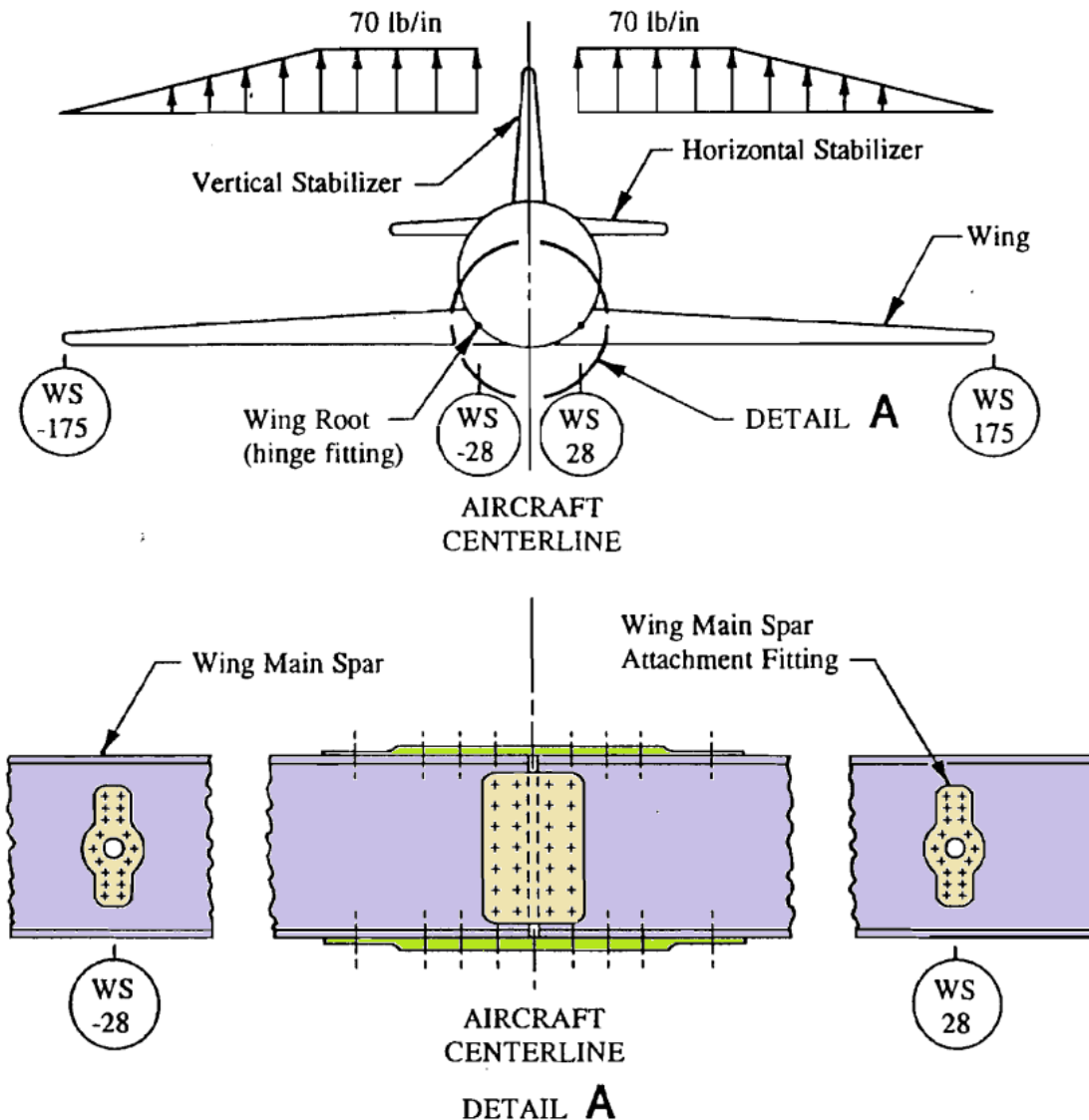
**FIGURE 1-32** Auxiliary wing spar with accompanying axial, shear, and bending moment diagrams constructed.

Now, starting at the left end of the axial diagram, the maximum compression axial load for this beam occurs between points A and C, while the maximum axial tension load occurs from points C to F, and zero axial load acting from F to G. To distinguish between compression and tension axial loads, the axial diagram is conveniently separated by the baseline shown in the figure. What is particularly interesting to note here about this diagram is that even though an applied axial load of

800 lb was externally applied locally to the structure at point C, the auxiliary wing spar internally only feels a maximum axial compression load of 500 lb and a maximum axial tension load of 300 lb. This discrepancy that exists between the external loaded structure and its corresponding internal loads picture of the loaded structure. Without it, or to do otherwise, would certainly be unacceptable, which would ultimately lead to the overdesign of the basic structure.

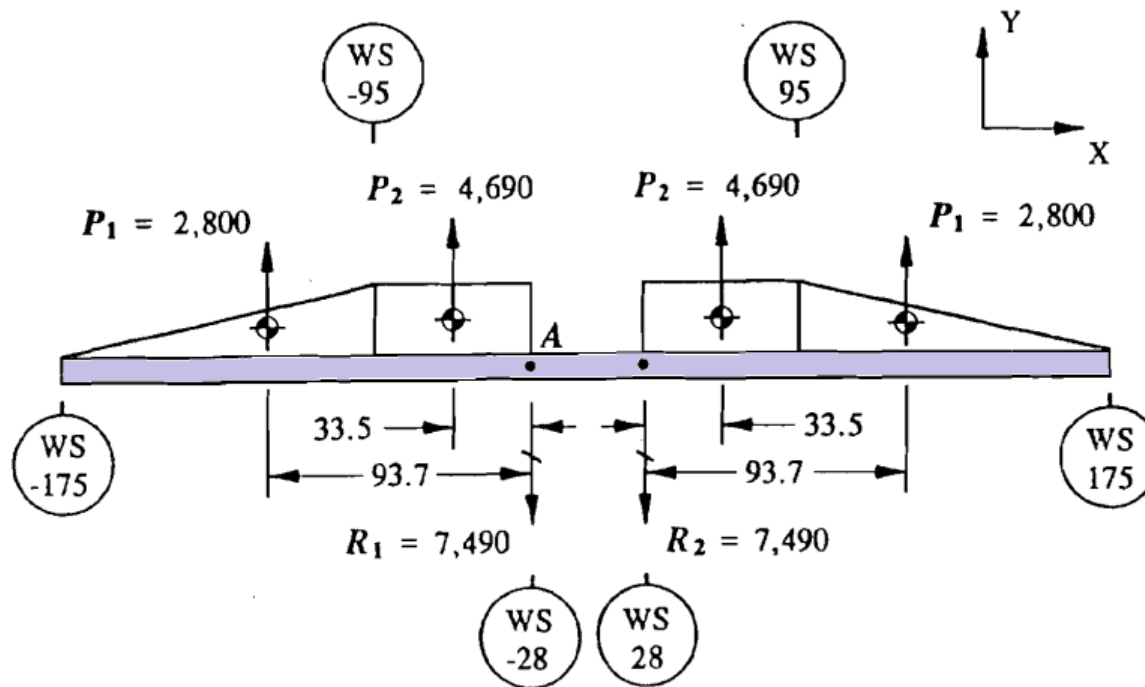
## Example 1-4:

Construct the shear and bending moment diagrams for the main spar.



**FIGURE 1-33** Simplified aerodynamic wing up-bending loads.

Solution:



**FIGURE 1-34** Wing root reactions indicated on a free-body diagram of the main spar.

$$P_1 = \frac{1}{2} wL = \frac{1}{2} (70)(175 - 95) = 2,800 \text{ lb}$$

$$P_2 = wL = 70(95 - 28) = 4,690 \text{ lb.}$$

Solution:

$$\Sigma F_y = 0 \quad \text{and} \quad \Sigma M_z = 0.$$

In the example considered, this gives:

$$\Sigma F_y = 0, \quad P_1 + P_2 - R_1 - R_2 + P_2 + P_1 = 0.$$

Substituting for  $P_1$  and  $P_2$ ,

$$\begin{aligned} 2,800 + 4,690 - R_1 - R_2 + 4,690 + 2,800 &= 0 \\ -R_1 - R_2 &= -14,980. \end{aligned}$$

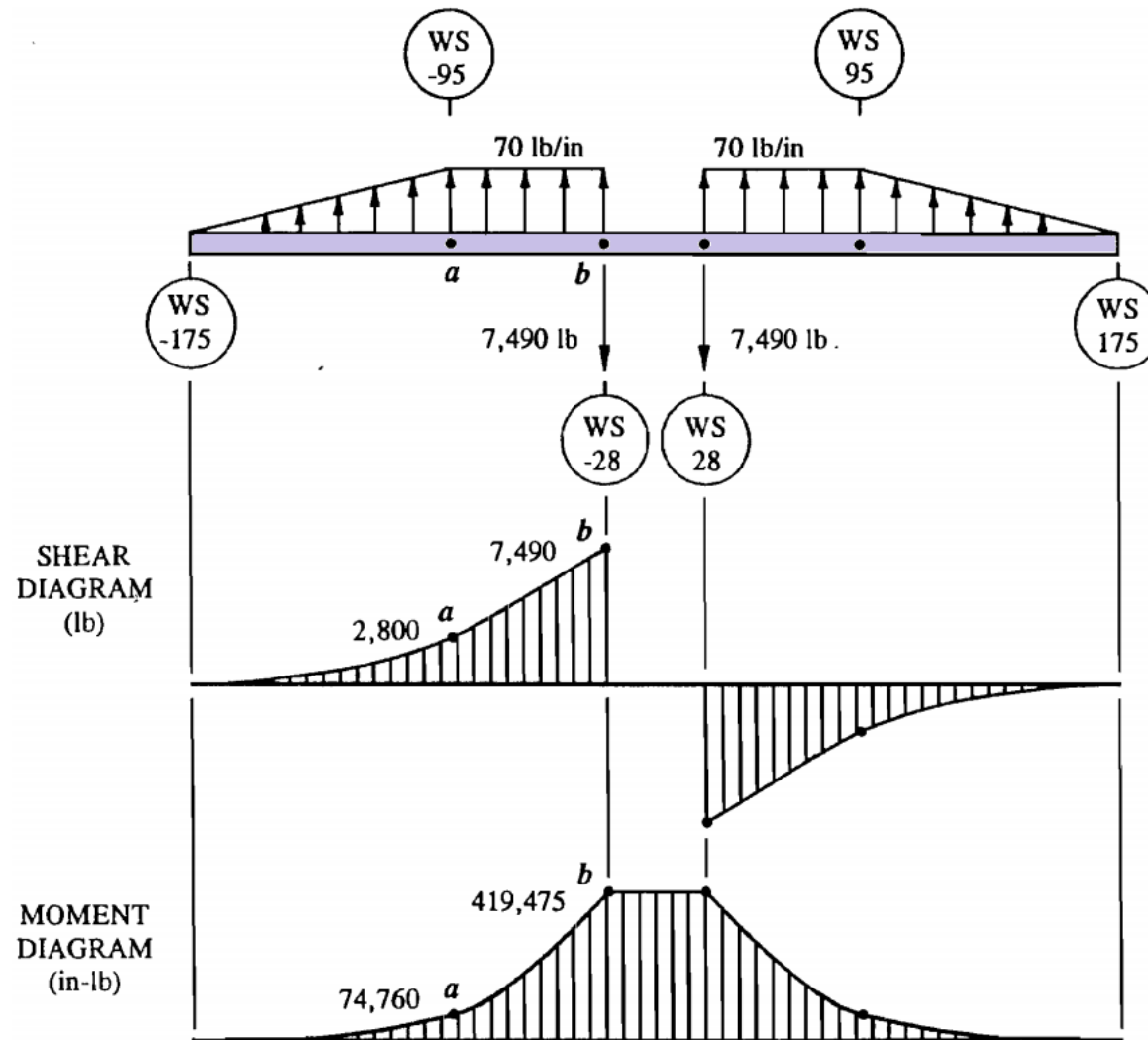
From  $\Sigma M_z = 0$  at point A,  $R_2$  is obtained directly:

$$\begin{aligned} \Sigma M_A = 0, \quad (+) \quad & 4,690(33.5) + 2,800(93.7) + R_2(56.0) \\ & - 4,690(89.5) - 2,800(149.7) = 0 \\ & R_2 = 7,490 \text{ lb.} \end{aligned}$$

Hence, from  $\Sigma F_y = 0$ ,  $R_1$  is

$$\begin{aligned} -R_1 - 7,490 &= -14,980 \\ R_1 &= 7,490 \text{ lb.} \end{aligned}$$

Solution:



**FIGURE 1-35** Completely balanced main spar, (a) shear diagram, (b) bending moment diagram.



## Solution:

Internal loads at point *a*, Fig. 1-36:

$$\Sigma F_x = 0, \quad P = 0$$

$$\Sigma F_y = 0, \quad 2,800 - V = 0, \quad V = 2,800 \text{ lb}$$

$$\Sigma M_a = 0, \quad \curvearrowleft 2,800(26.7) - M_a = 0, \quad M_a = 74,760 \text{ in-lb.}$$

Internal loads at point *b*, Fig. 1-37:

$$\Sigma F_x = 0, \quad P = 0$$

$$\Sigma F_y = 0, \quad 2,800 + 4,690 - V = 0, \quad V = 7,490 \text{ lb}$$

$$\Sigma M_b = 0, \quad \curvearrowleft 2,800(93.7) + 4,690(33.5) - M_b = 0$$

$$M_b = 419,475 \text{ in-lb}$$

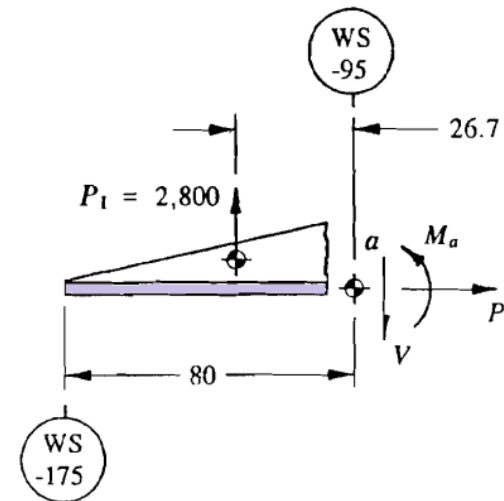


FIGURE 1-36 Internal loads at an exposed section of beam at point *a*.

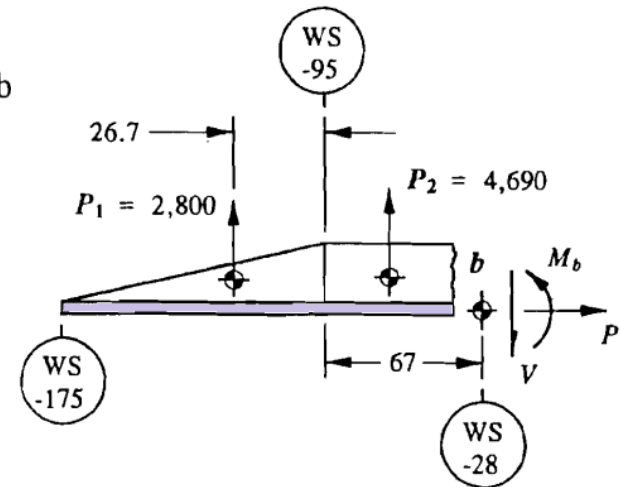
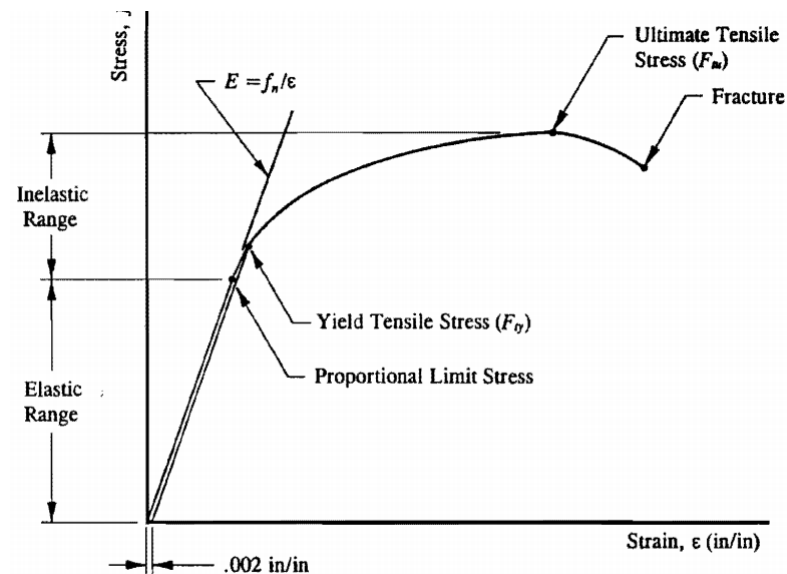


FIGURE 1-37 Internal loads at an exposed section of beam at point *b*.

This method states that the resultant effect of several applied forces on a structure is the numerical sum of those effects when the applied forces are acting separately.

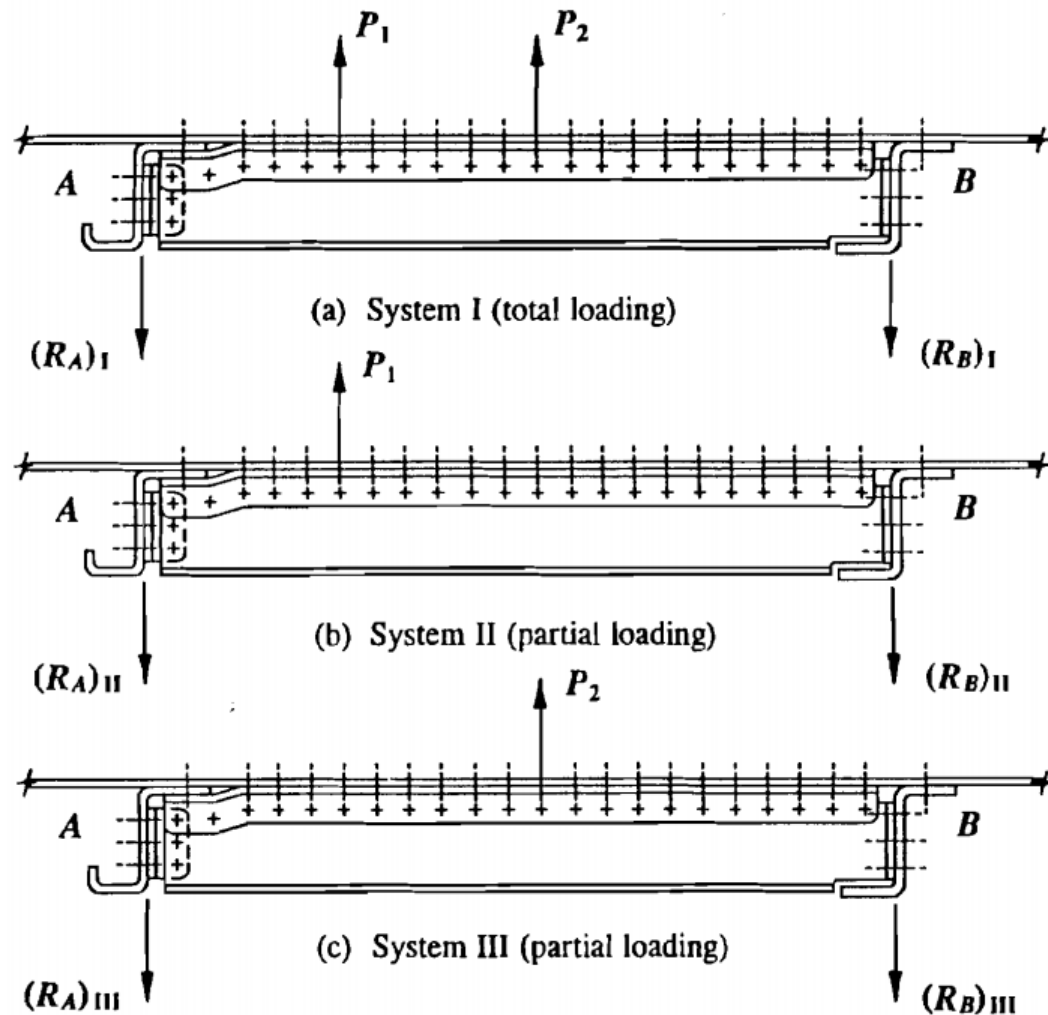
## Limitations:

- Deflections of the structure must be directly proportional to the applied loads
- Stresses must be below the proportional limit of the material



Inelastic range = stresses that occur above the proportional limit stress.  
Elastic range = stresses that occur below the proportional limit stress.

# PRINCIPLE OF SUPERPOSITION



$$\text{System I} = \text{System II} + \text{System III.}$$

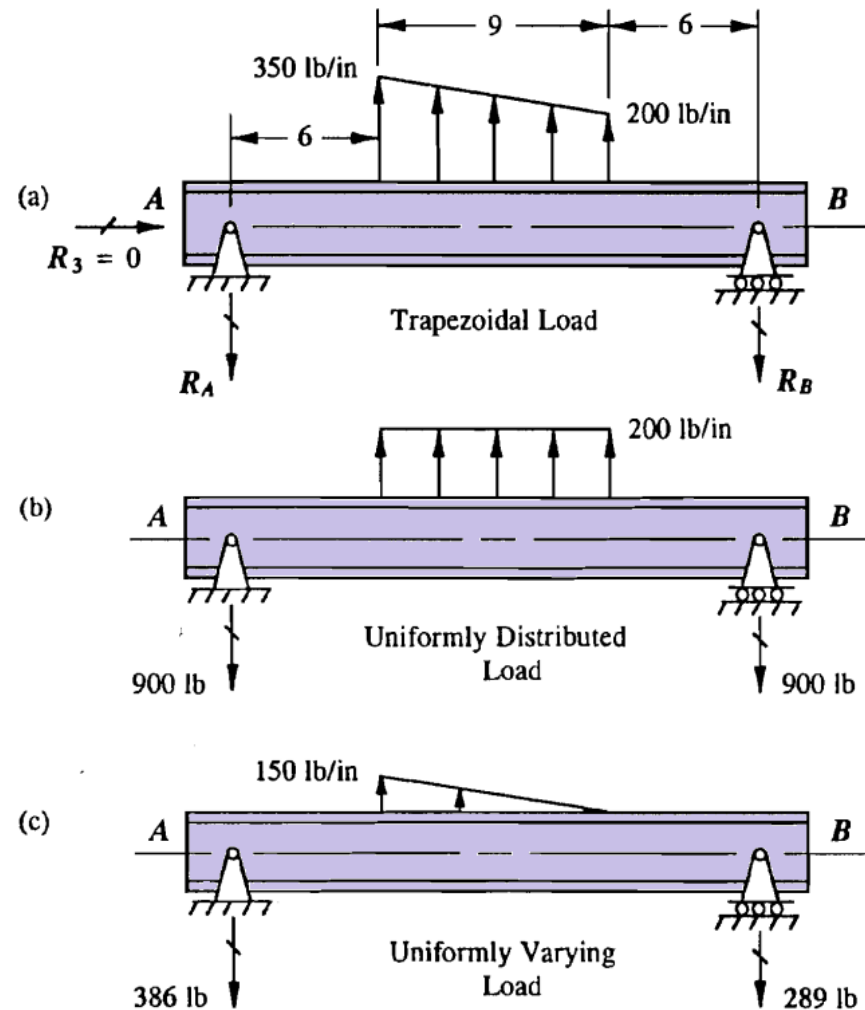
$$(R_A)_I = (R_A)_{II} + (R_A)_{III}$$

$$(R_B)_I = (R_B)_{II} + (R_B)_{III}$$

**FIGURE 1-38** Principle of superposition of reaction forces.

## Example 1-5:

Find the internal forces 5.0 in. From the support at A for trapezoidal loading given. Moreover, show the internal forces on the beam segments to each side of the proposed section cut.

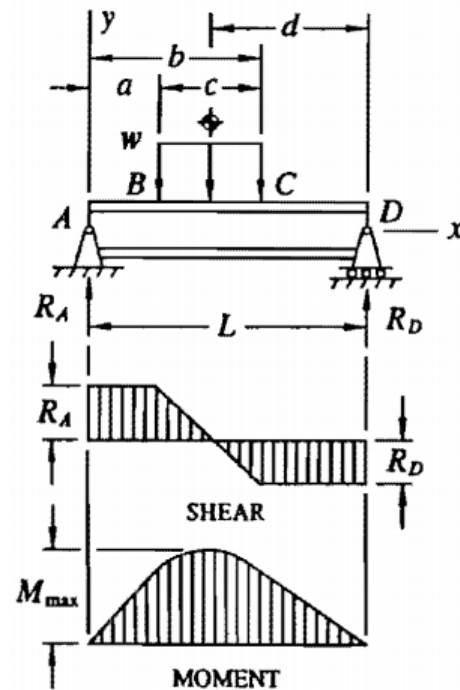


**FIGURE 1-39** (a) Trapezoidal loaded beam. Separation of the trapezoidal loading into two separate loading types: (b) the uniformly distributed load and (c) the uniformly distributed varying load.

Solution:

System 2 (App. D Table A-1):

**Case 12 BEAM SIMPLY-SUPPORTED AT BOTH ENDS—Uniform load partially distributed over span.**



REACTIONS

$$R_A = \frac{wcd}{L} \quad R_D = \frac{1}{2} \frac{wc}{L} (2a + c)$$

SHEARS

$$V = -R_A \quad (A \text{ to } B)$$

$$V = -\frac{w}{L}(cd - Lx + aL) \quad (B \text{ to } C)$$

$$V = R_D \quad (C \text{ to } D)$$

BENDING MOMENTS

$$M = -R_A x \quad (A \text{ to } B)$$

$$M = -R_A x + \frac{1}{2} w (x - a)^2 \quad (B \text{ to } C)$$

$$M = -R_A x + \frac{1}{2} wc (2x - a - b) \quad (C \text{ to } D)$$

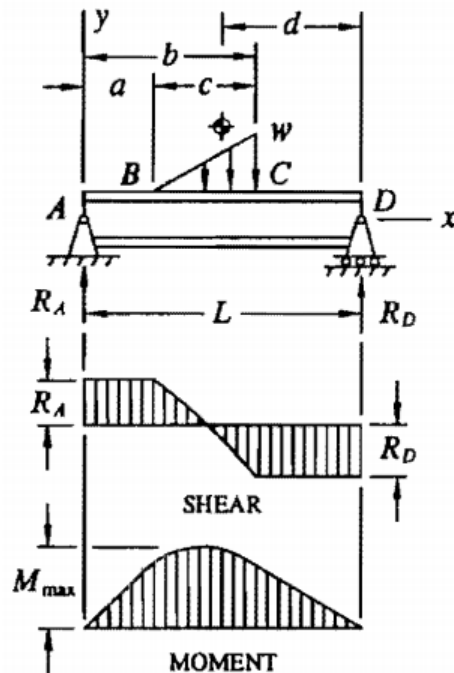
$$M_{max} = -R_A \left( a + \frac{R_A}{2w} \right) \quad \text{at } x = a + \frac{R_A}{w}$$

$$R_A = R_B = \frac{200 \cdot 9 \cdot 10.5}{21} = 900 \text{ lb.}$$

Solution:

System 3 (App. D Table A-1):

**Case 14** BEAM SIMPLY-SUPPORTED AT BOTH ENDS—Uniform load partially distributed over span.



REACTIONS

$$R_A = \frac{1}{2} \frac{wcd}{L} \quad R_D = \frac{1}{2} \frac{wc}{L} (L - d)$$

SHEARS

$$V = -R_A \quad (A \text{ to } B)$$

$$V = -R_A + \frac{1}{2} \frac{w}{c} (x - a)^2 \quad (B \text{ to } C)$$

$$V = R_D \quad (C \text{ to } D)$$

BENDING MOMENTS

$$M = -R_A x \quad (A \text{ to } B)$$

$$M = -R_A x + \frac{w(x - a)^3}{6c} \quad (B \text{ to } C)$$

$$M = -R_A x + \frac{1}{6} wc(3x - a - 2b) \quad (C \text{ to } D)$$

$$M_{max} = -R_A \left[ a + \frac{2}{3} c \sqrt{\frac{d}{L}} \right] \quad \text{at } x = a + c \sqrt{\frac{d}{L}}$$

$$R_A = \frac{1}{2} \cdot \frac{150 \cdot 9}{21} (21 - 9) \cong 386 \text{ lb.} \quad R_B = \frac{1}{2} \cdot \frac{150 \cdot 9 \cdot 9}{21} \cong 289 \text{ lb.}$$

Solution:

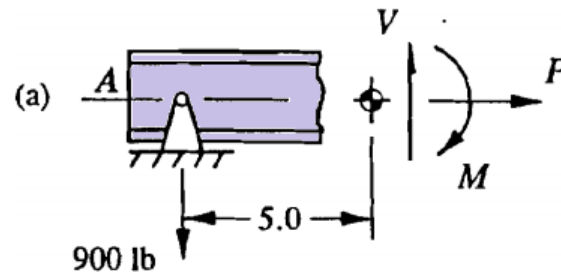
$$\text{System I} = \text{System II} + \text{System III}$$

Hence,

$$R_A = 900 + 386 = 1,286 \text{ lb}$$

and

$$R_B = 900 + 289 = 1,189 \text{ lb.}$$

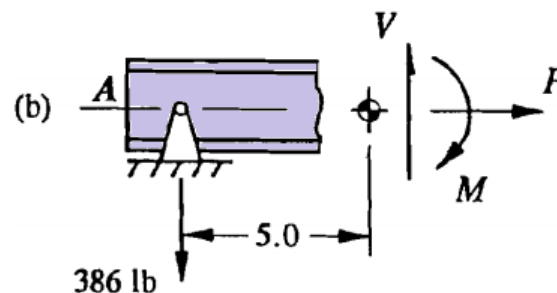


*From statics:*

$$V = 900 \text{ lb}$$

$$P = 0 \text{ lb}$$

$$M = 900(5.0) = 4,500 \text{ in-lb.}$$



*From statics:*

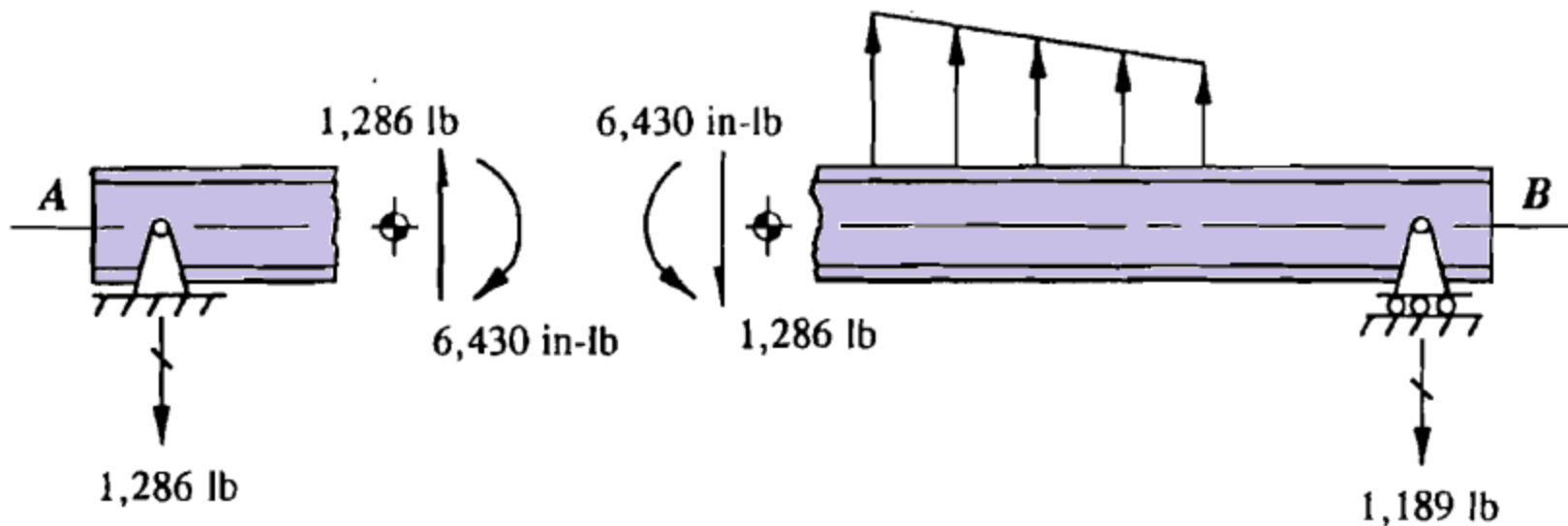
$$V = 386 \text{ lb}$$

$$P = 0 \text{ lb}$$

$$M = 386(5.0) = 1,930 \text{ in-lb.}$$

**FIGURE 1-40** Internal loads at exposed sections of beam: (a) for the uniformly distributed load and (b) for the uniformly distributed varying load.

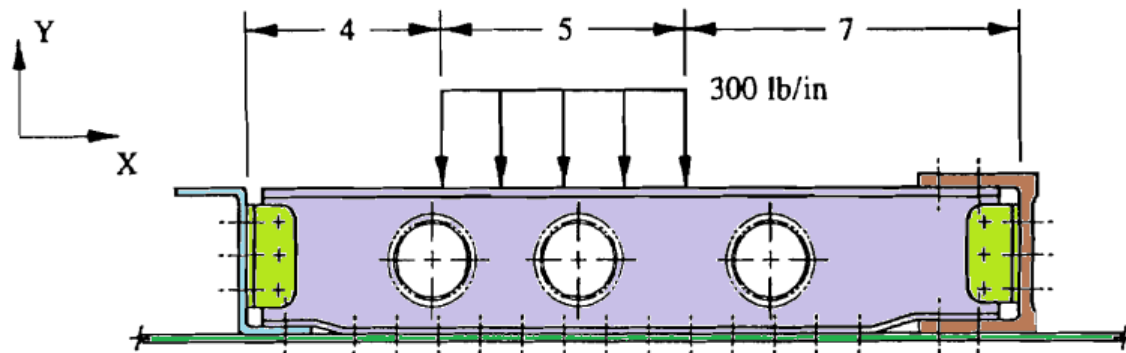
Solution:



**FIGURE 1-41** Final system of loads for the two separate loading types.



## Example 1-6:



**FIGURE 1-42** Intercostal loaded by a uniformly distributed load.

Determine the support reactions using the formula given in Appendix D, Table A-1. Additionally, indicate the maximum shears and bending moments on properly drawn shear and bending moment diagrams, respectively.