



AEE 461 Design of Aircraft Structures

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Lecture #5

Axial and Bending Members – cont'd

Example 2-2:

To provide additional stiffness to the engine air-inlet duct skin, it is proposed that four (4) typical stiffeners equally spaced between Frame Stations 495.0 and 535.0 be used. Such an arrangement is shown in below, where, based on the duct geometry given, the stiffeners are of unequal lengths. If the combined pressure of the internal cabin pressure and external air pressure on the surface of the duct is 39.3 psi ultimate acting outward from the center of the fuselage, what are the maximum fiber stresses for the most critical stiffener? To simplify the solution, neglect the distribution of pressure loading that is carried by the longitudinal edge members. Assume that the air-inlet duct skin rivets are .159 inch in diameter

2.3 Bending of Beams in One Plane

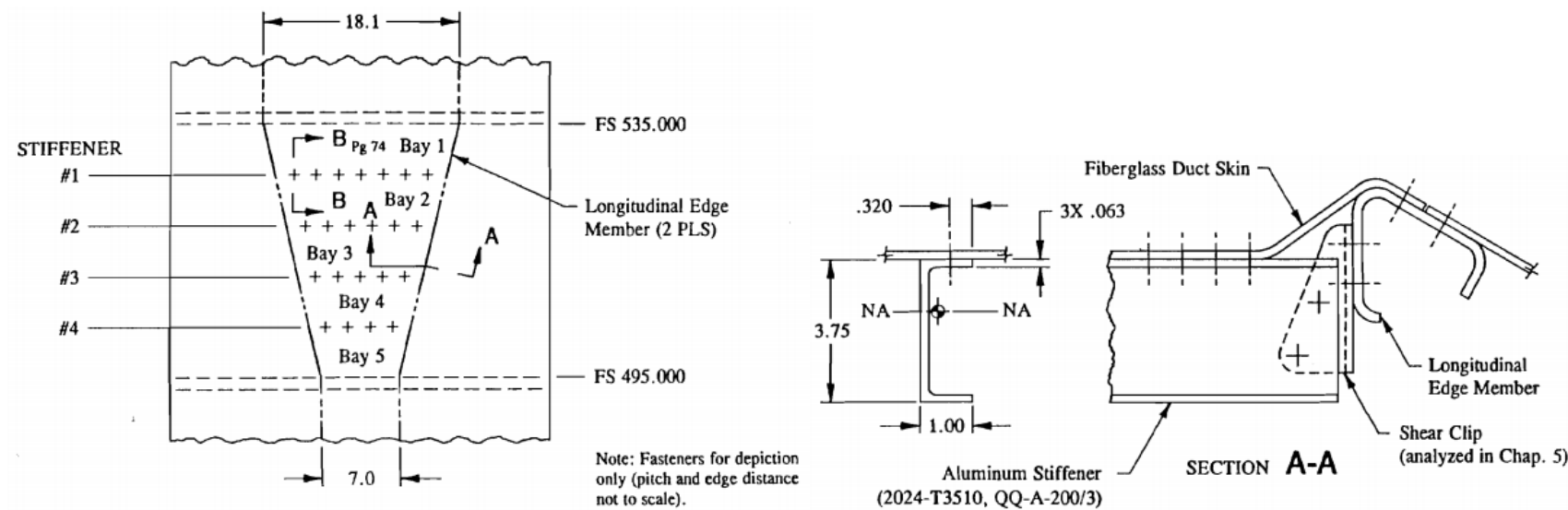


FIGURE 2-10 Engine air-inlet duct shown with typical stiffeners equally spaced.

2.3 Bending of Beams in One Plane

The first step in the solution of this problem is to determine the amount of pressure loading that is carried by each stiffener. This is done conveniently by proportioning one-half ($\frac{1}{2}$) of the pressure loading acting in each of the air-inlet duct bays to each of the adjacent transverse structural members: the stiffeners and frames, as shown in Fig. 2-11(a), provide this function. To accomplish this mathematically, the pressure loading is idealized as a distributed load w (lb/in) acting along the length of each stiffener or frame, and their numerical values are computed by taking the product of the pressure p and the effective width b of loading considered, or

$$w = p \cdot b \text{ (see Table 1-1, Case 3 in Sec.1.3)}$$

$$w = 39.3(8.0) = 314.4 \text{ lb/in. ultimate}$$

2.3 Bending of Beams in One Plane

In Chapter 5, a method of isolating and analyzing shear clips will be presented. For the time being, let us assume that the stiffener shear clips are of adequate strength to transfer the shear reactions R_1 and R_2 indicated in the figure. The maximum bending moment for any stiffener occurs at midspan, and is represented by the following equation:

$$M = \frac{wL^2}{8} = \frac{314.4(15.9)^2}{8} = 9.935 \text{ in-lb, Ultimate}$$

where from the geometry given in this problem, the length of stiffener #1 from A to B is 15.9 in. Based upon principles previously learned in Chapter 1, the direction of this moment is easily figured out: tension on the outer flange (skin side) and compression on the inner flange side.

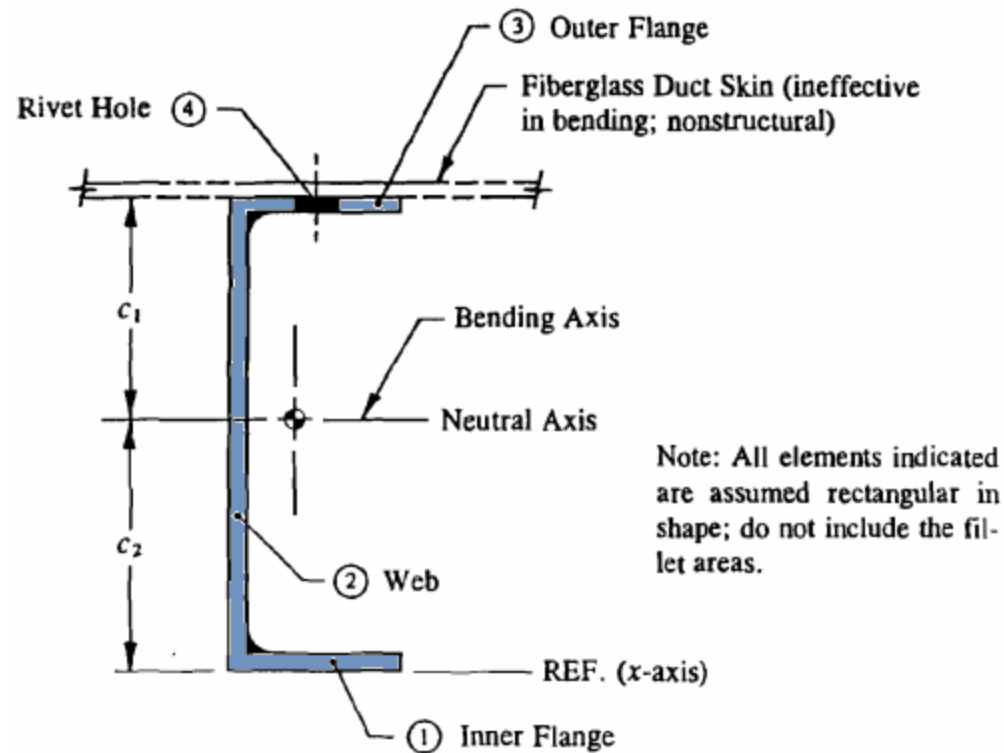
From the proportionality of terms in this expression, it is clear that the longer stiffener is more critical. This is true because the ultimate strength of a member is usually based on its maximum design moment, which, in this case, occurs at center span for the longer stiffener.

In order to apply the flexure formula ($f_b = \pm Mc/I_{na}$), the moment of inertia I_{na} around the neutral axis of the cross-sectional area of the stiffener must be computed. This computation will also establish the maximum c distances measured from the neutral axis to the location on the surface where the maximum compression and tension stresses occur.

2.3 Bending of Beams in One Plane

Since fiberglass is structurally many times weaker than aluminum, the fiberglass duct skin will not be considered as an effective member in providing any appreciable bending resistance with the attached stiffener. The duct skin does however provide the physical restraint to prevent the stiffener from twisting out-of-plane. The skin actually forces bending to occur about an axis parallel to its skin line. Of course, this axis must also pass through the neutral axis of the effective stiffener area. Note, in particular, that if the fiberglass duct skin was not present to stabilize the stiffener, bending would then occur around the principal axes of the cross-sectional area. With no axis of symmetry, the principal axes of the section would have to be computed (see Appendix G, "Calculation of Principal Moments of Inertia"). And then the flexure formula would apply around these axes.

2.3 Bending of Beams in One Plane



<i>Element#</i>	<i>+ or -</i>	<i>b</i>	<i>h</i>	<i>y</i>	<i>A</i>	<i>Ay</i>	<i>Ay²</i>	<i>I₀</i>
1	+	1.000	0.063	0.031	0.063	0.002	0.000	0.000
2	+	0.063	3.264	1.875	0.228	0.428	0.803	0.250
3	+	1.000	0.063	3.718	0.063	0.234	0.871	0.000
4	-	0.159	0.063	3.718	-0.010	-0.037	-0.138	-0.000
Total					0.344	0.627	1.536	0.250

2.3 Bending of Beams in One Plane

Cross-sectional properties:

$$Y_{cg} = \frac{\sum Ay}{\sum A} = \frac{.627}{.344} = 1.823 \text{ in}$$

$$I_{cg} = \sum I_o + \sum Ay^2 - Y_{cg}(\sum Ay)$$
$$I_{cg} = .250 + 1.536 - 1.823(.627) = .643 \text{ in}^4.$$

$$Y_{na} = Y_{cg} = 1.823 \text{ in}$$

$$I_{na} = I_{cg} = .643 \text{ in}^4.$$

Stress at skin side:

$$f_t = + \frac{M c_1}{I_{na}}$$
$$f_t = \frac{9,935(3.75 - 1.823)}{.643} = 29,774 \text{ psi ult}$$

Stress at inner flange:

$$f_c = - \frac{M c_2}{I_{na}}$$
$$f_c = - \frac{9,935(1.823)}{.643} = - 28,167 \text{ psi ult}$$

Ultimate tension failure of the outer flange:

$$\text{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{57,000}{29,774} - 1 = + 0.91 \text{ or } 91\%.$$

Yield compression failure of the inner flange:

$$\text{M.S.} = \frac{F_{cy}}{f_c} - 1 = \frac{34,000}{28,167} - 1 = + 0.21 \text{ or } 21\%.$$

Note that: This solution has purposely omitted the consideration of the effects of crippling failure of the inner flange of the critical stiffener. Crippling is a condition of buckling whereby a member under compression loading can fail by virtue of its instability at a compression stress level which may be lower than the indicated yield compression value of the material F_{cy} . To avoid the possibility of this type of failure occurring unexpectedly or prematurely, the crippling allowable F_{cc} of a compression member must be investigated, and its value compared to the calculated compression stress f_c . Mathematically, this says that if $F_{cc} < F_{cy}$, the condition of instability is more critical and therefore governs the compression strength criterion of its design.

2.4 Method of Approximations Approach of Bending Members

The material of this section will establish a method of approximation to the analysis of members subjected to bending. Before we start our discussion, however, the engineer is referred to the couple-force relationship of a moment as it applies to a beam section, described mathematically in Appendix B by Eq. A-7 ($P_{couple} = M/H$). As a brief review of these principles, it was previously shown that the internal bending moment M that exists across a beam section can conveniently be replaced by an equivalent couple-force, P_{couple} , each acting in opposite directions and separated by a distance H between them. From this relationship, not only is the physical action of the beam mentally comprehended but by the use of Eq. 2-2 ($f_n = \pm P/A$), as it applies to each separate cap area, the following equations in terms of tension and compression stresses can be written:

Tension stress:
$$f_t = + \frac{P_{couple}}{A_{net}}$$

where $P_{couple} = M/H$, A_{net} = net cross-sectional area of the tension cap

Compression stress:
$$f_c = + \frac{P_{couple}}{A_{gross}}$$

where $P_{couple} = M/H$, A_{gross} = total cross-sectional area of the tension cap

2.4 Method of Approximations Approach of Bending Members

Once the application of these equations has been thoroughly learned, bending stress calculations, normally performed laboriously by using the flexure formula ($f_b = \pm Mc/I_{na}$), can now be alternatively done with rapidity and with a remarkable degree of accuracy. The reason this method is so quick, is that the bending stresses as defined by Eqs. 2-6 and 2-7 are entirely derived independent of moment of inertia calculations. The most time-consuming and boring part of a stress analysis solution of bending members is usually the calculation of this term.

Having applied the flexure formula to the solution of Example 2-1, page 67, let us now compare the results of that solution to the approximation method of this section. First, the distance H between cap centroids is estimated for Section B-B, as pictured in Fig. 2-13. The effective cap areas (shown cross-hatched) are written as follows:

$$A_{\text{upper cap}} = (1.0 + 1.0 - .159 - .063)(.063) = .112 \text{ in}^2$$

$$A_{\text{lower cap}} = (1.0 + 1.0 - .063)(.063) = .122 \text{ in}^2.$$

2.4 Method of Approximations Approach of Bending Members

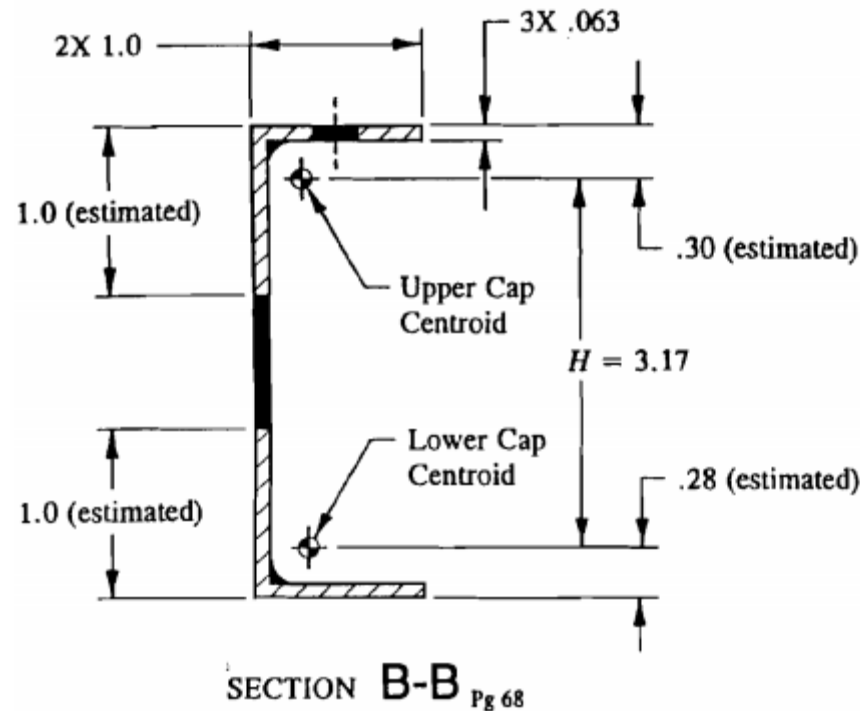


FIGURE 2-13 Distance between the upper and lower cap centroids is estimated for Section B-B.

Since our solution is intended to be approximate in nature, just guess at the distance H between cap centroids of the cross-sectional area. From the sketch, it is apparent that

$$H = 3.75 - .300 - .280 = 3.17 \text{ in.}$$

2.4 Method of Approximations Approach of Bending Members

The moment M is then replaced by an equivalent couple-force between the cap centroids of Section B-B using Eq. A-7 of Appendix B. Thus, in the example considered, this gives:

$$P_{\text{couple}} = \frac{M}{H} = \frac{9,935}{3.17} = 3,134 \text{ lb.}$$

Next, Eqs. 2-6 and 2-7 are applied to each designated cap area.

$$f_t = + \frac{P_{\text{couple}}}{A_{\text{net}}} = \frac{3,134}{.112} = 27,982 \text{ psi (tension outer flange)}$$

$$f_c = - \frac{P_{\text{couple}}}{A_{\text{gross}}} = - \frac{3,134}{.122} = -25,689 \text{ psi (compression inner flange).}$$

The stress results obtained from the approximation method here and the flexure formula (Example 2-1) are summarized below:

Member	Approximation Method (M/H)	Flexure Formula (Mc/I_{na})	% Deviation
Outer Flange	27,982	29,774	6.0
Inner Flange	- 25,689	- 28,167	8.8

The internal axial forces that sometimes occur simultaneously with internal bending moments can also be approximated - the axial forces are distributed to each designated cap by a ratio of their individual cap areas to the total area of the section considered. The final results are obtained when these values are superimposed with the bending loads of this section. This gives the combined effects desired. Using the following expression, the magnitude of force of each cap can be found:

$$P_{\text{cap}} = P \left[\frac{A_{\text{cap}}}{A_{\text{total}}} \right]$$

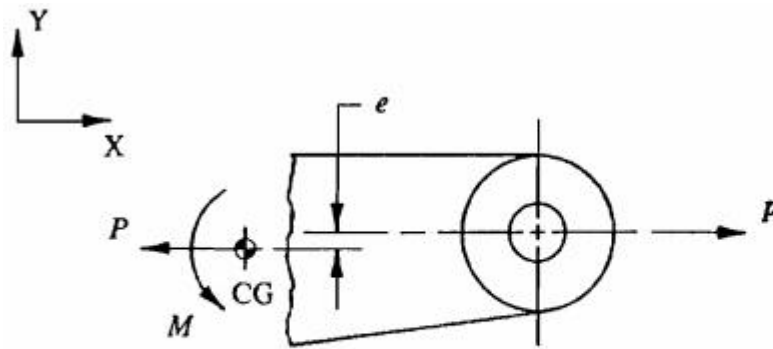
where: P = internal axial force (from statics)

A_{cap} = individual cap area

A_{total} = total beam cross-sectional area.

Note that: The equation above is derived from axial stiffness consideration

2.5 Axial and Bending Stresses of Beams that Bend in One Plane



$$\begin{aligned}\Sigma F_x &= 0, & P &= p \\ \Sigma M_{cg} &= 0, & \curvearrowright pe - M &= 0, & M &= pe.\end{aligned}$$

FIGURE 2-14 Internal bending moment is developed across the section area due to an eccentrically applied axial force.

For illustrative purposes, when an externally applied axial force does not act through the centroidal axis of the cross-sectional area of a beam, as shown in Fig. 2-14, the eccentricity of force e will cause a bending stress distribution by virtue of the internal bending moment M which is developed across the section area. As a good design practice, to limit the increase in overall structural weight of a final design, all member eccentricities should be avoided or at least minimized, and a deliberate and careful consideration made to eliminate their occurrences. To find the internal forces at this section, an imaginary section cut is made through the member and the equations of statics applied by the usual methods of internal loads analysis.

2.5 Axial and Bending Stresses of Beams that Bend in One Plane

Physically, to produce the actual stresses on the cross-sectional area, the internal force P must be represented at the centroid of the cross-sectional area of the member, while the bending moment M establishes its stress distribution with respect to the neutral axis of the section area. Then, the appropriate stress formulas are each 'applied separately to determine the internal stresses which are caused by these internal forces, as shown in Fig. 2-15.

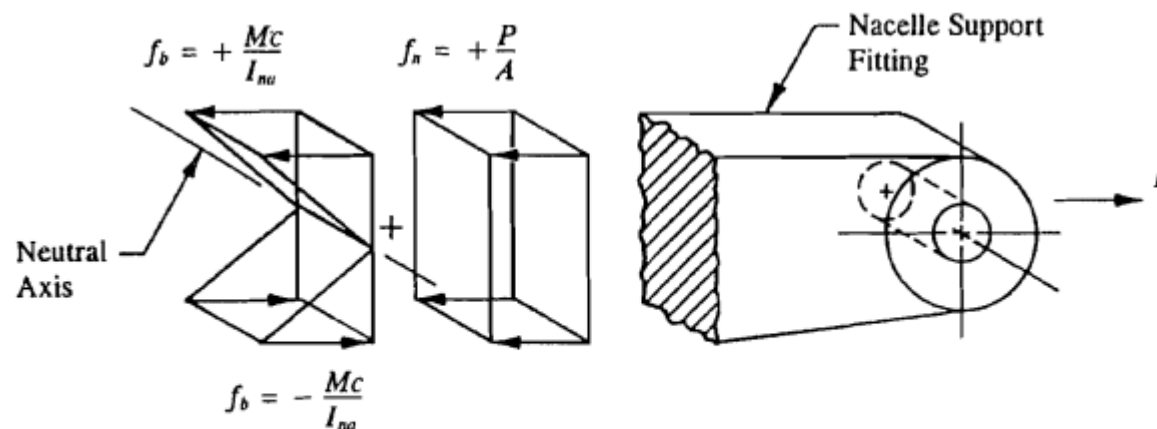


FIGURE 2-15 Axial and bending stresses produce similar stresses normal to the plane of the section area.

$$f = \pm \frac{P}{A} \pm \frac{Mc}{I_{na}}.$$

2.5 Axial and Bending Stresses of Beams that Bend in One Plane

Example 2-3:

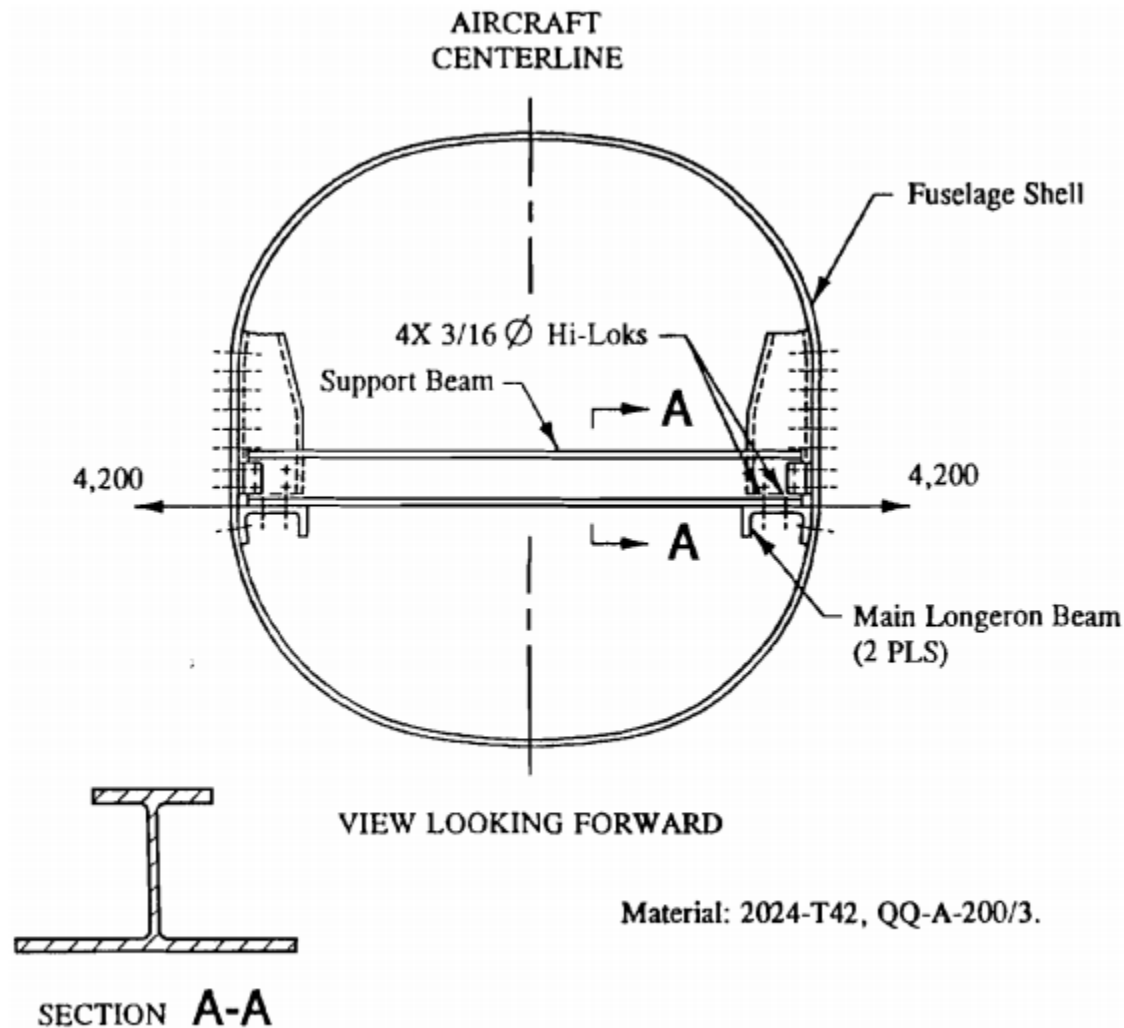


FIGURE 2-17 Support beam spanning the fuselage shell.

Example 2-3 – cont'd:

Instead of redesigning the main longeron beams of an increased gross-weight model aircraft, it was proposed to provide an additional support for these members by spanning a support beam completely across the fuselage shell, as shown in Fig. 2-17. The additional support will have the desired effect of decreasing the peak internal loads for the longerons.

- (a) If the support beam reacts 4,200 lb ultimate load through its lower flanges, what maximum extreme fiber stresses will occur for this member?
- (b) Write the margins of safety in compression and tension.

2.5 Axial and Bending Stresses of Beams that Bend in One Plane

Solution:

Since the applied loads do not lie in the same plane with the centroidal axis of the beam, it should be clear that this effect will produce an eccentricity of load e which in turn will induce a constant internal bending moment $M = 4,200(e)$ along the entire length of the support beam. Once the eccentricity is found, the magnitude of this moment can be computed. To determine these unknown quantities, the usual methods of analysis are employed: the support beam is completely isolated from the fuselage shell and the corresponding structure free bodied. This is shown in Fig. 2-18.

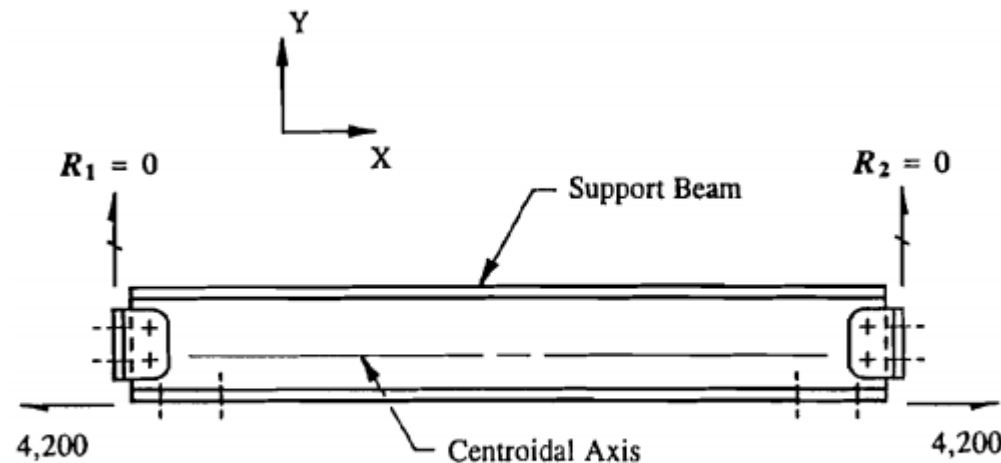


FIGURE 2-18 Support beam is isolated from the fuselage shell and free-bodied.

Solution - cont'd:

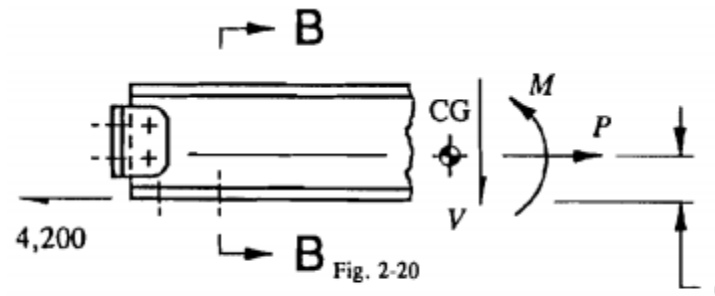


FIGURE 2-19 Internal loads represented on an exposed section of beam.

Next, the magnitude of the internal loads are investigated at arbitrary points along the entire length of the support beam, see Fig. 2-19 for the representation of these loads on an exposed section of beam. If we apply the method of sections approach to internal loads analysis at this section, we obtain the following values for V , P , and M :

$$\Sigma F_y = 0, \quad V = 0$$

$$\Sigma F_x = 0, \quad P - 4,200 = 0, \quad P = 4,200 \text{ lb ult}$$

$$\Sigma M_{cg} = 0, \quad \curvearrowright \quad 4,200(e) - M = 0, \quad M = 4,200(e) \text{ in-lb ult.}$$

Solution - cont'd:

From these equations, it can be seen that the internal loads P and M are constant anywhere between the supported ends of the beam, but not specifically including the joints themselves.

Element	b	h	y	A	Ay	Ay^2	I_o
1	2.300	0.080	0.040	0.184	0.007	0.000	0.000
2	0.080	1.200	0.680	0.096	0.065	0.044	0.012
3	0.190*	0.080	0.040	-0.015	-0.001	-0.000	-0.000
4	0.190*	0.080	0.040	-0.015	-0.001	-0.000	-0.000
5	0.500	0.080	1.320	0.040	0.053	0.070	0.000
Total				0.290	0.123	0.114	0.012

* The nominal pin or shank diameter is normally used for bolts in a tension joint (see specific values given in Table 8.1.5.2(n) of MIL-HDBK-5F for this fastener).

$$Y_{cg} = \frac{\sum Ay}{\sum A} = \frac{.123}{.290} = .424 \text{ in}$$

2.5 Axial and Bending Stresses of Beams that Bend in One Plane

Solution - cont'd:

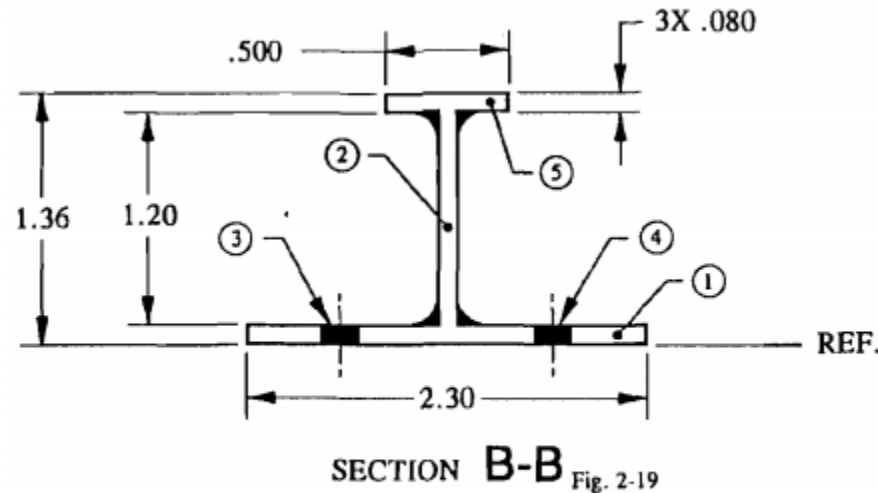


FIGURE 2-20 Support beam cross-sectional area. Neglect the fillet areas of the extruded I-beam.

$$I_{cg} = \Sigma I_o + \Sigma Ay^2 - Y_{cg}(\Sigma Ay) = .012 + .114 - .424(.123) = .074 \text{ in}^4.$$

The eccentricity of force e corresponds to the value Y_{cg} above, or

$$e = Y_{cg} = .424 \text{ in}$$

and, from $M = 4,200(e)$, we now obtain:

$$M = 4,200(.424) = 1,781 \text{ in-lb ult.}$$

Solution - cont'd:

Thus, the elastic stresses at the extreme fibers caused by the combination of axial force P and bending moment M are determined:

Ultimate tension stress of the lower flange:

$$f_t = \pm \frac{P}{A} \pm \frac{Mc}{I_{na}}$$
$$f_t = + \frac{4,200}{.290} + \frac{1,781(.424)}{.074} = 14,483 + 10,205 = + 24,688 \text{ psi.}$$

Ultimate compression stress of the upper flange:

$$f_c = \pm \frac{P}{A} \pm \frac{Mc}{I_{na}}$$
$$f_c = + \frac{4,200}{.290} - \frac{1,781(1.36 - .424)}{.074} = 14,483 - 22,527 = - 8,044 \text{ psi.}$$

Solution - cont'd:

The mechanical properties of 2024-T42 aluminum alloy extrusion (Q-QA-200/3) in the longitudinal direction are as follows: $F_{tu} = 57$ ksi, $F_{ty} = 38$ ksi, and $F_{cy} = 38$ ksi (see mechanical-property table for this material in MIL-HDBKSF). If we compare these allowable values with those stresses calculated, clearly, we find that tension and compression stresses are well below yield allowable values. And this necessary condition therefore confirms or validates our original assumption of elastic stress behavior of the support beam. The margins of safety are then computed as follows:

Ultimate tension failure of the lower flange:

$$\text{M.S.} = \frac{F_{tu}}{f_t} - 1 = \frac{57,000}{24,688} - 1 = +1.31 \text{ or } 131\%.$$

Yield compression failure of the upper flange:

$$\text{M.S.} = \frac{F_{cy}}{f_c} - 1 = \frac{38,000}{8,044} - 1 = + \text{HIGH!!}$$