



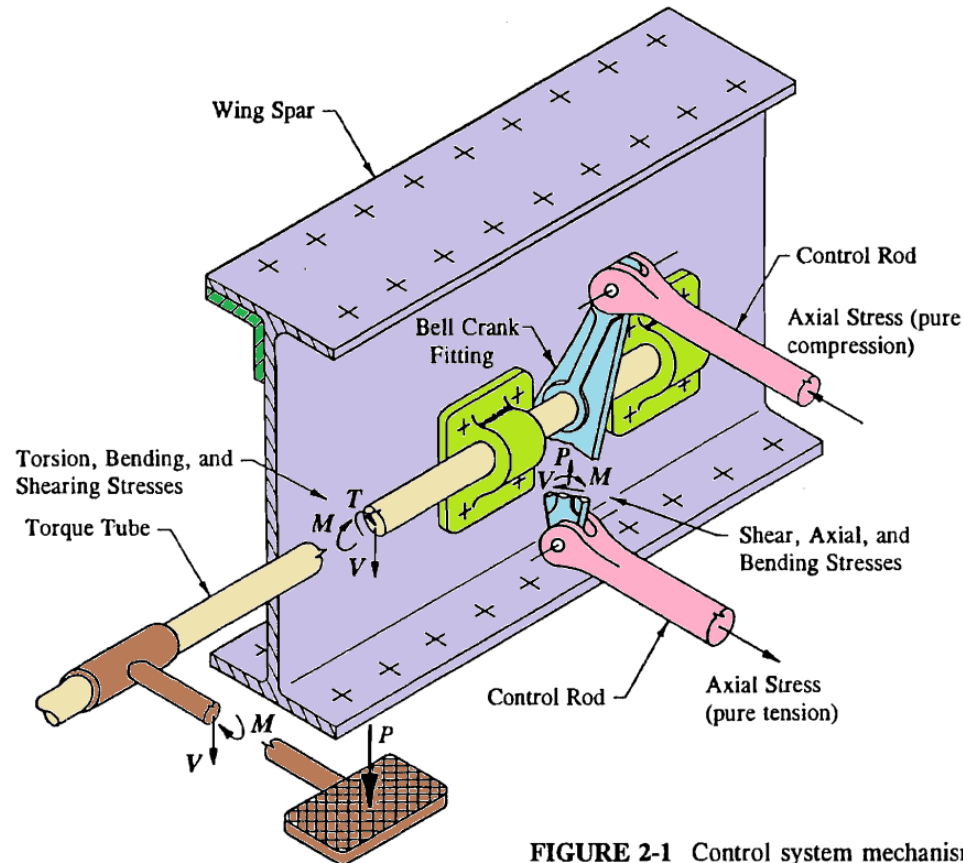
# AEE 461 Design of Aircraft Structures

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## Lecture #4 Axial and Bending Members

## 2.1 Introduction

Stress is a measure of internal forces in a body between its particles.  
Stress is the force per unit area of a body that tends to cause it to change shape.  
Stress is the average force per unit area that a particle of a body exerts on an adjacent particle, across an imaginary surface that separates them.



**FIGURE 2-1** Control system mechanism showing the internal loads members are subjected to.

## 2.1 Introduction

In all phases of stress analysis, the following questions must be answered to achieve a complete and dependable structural design solution:

1. What magnitude of force can be applied to the structure or to its component parts before failure occurs?
2. How much deformation is acceptable under prescribed loading conditions?
3. What is the most efficient size member required for the structure to withstand these forces?

If the component force represents the force perpendicular to the section, then it is called «normal»

**Table 2-1 Stress Formulas**

Type of Stress	Formula	Reference
Normal Stress	$f_n = \pm \frac{P}{A}$	Ch.2, p.57
Bending Stress	$f_b = \pm \frac{Mc}{I}$	Ch.2, p.63
Shear Stress	$f_s = \frac{VQ}{I_{na}t}$	Ch.4, p.227

## 2.1 Introduction

Applied stresses can be calculated via using the equations given in Table 2-1. What about the Allowable stresses?

Allowable stresses obtained from tests can be found in MIL-HDBK-5. Without this we cannot say anything about structure whether it is failing or not.

The «margin of safety» means that the margin required in order to insure safety. Its numerical value provides a means by which the engineer can communicate to his associates and others of his analytical findings relating to his predictions of structural failure as a result of some prescribed physical characteristic of the structure, such as ultimate failure stresses, fatigue considerations, instability requirements, deformation criteria, etc. Therefore, the following expression is formulated:

$$\text{Margin of Safety} = M.S. = \frac{\text{allowable stress}}{\text{applied stress}} - 1$$

Note that; where possible, the engineer should strive to optimize his designs by maintaining «zero» margins of safety. Or, at least, to avoid premature failure, he must realize slightly «positive» values for each of his predictions or margins of safety.

## 2.2 Axial Compression and Tension Stresses

The following equation to be used in order to obtain axial compression and tension normal stresses

$$f_n = \pm \frac{P}{A}$$

where  $P$  = internal axial force acting perpendicular (or normal) to a section and passing through the centroid of the cross-sectional area of a beam

$A$  = effective cross-sectional area.

The following predictions of tension and compression failures of the material can be written:

Ultimate Tension Failure:

$$M.S. = \frac{F_{tu}}{f_t} - 1$$

where  $F_{tu}$  = ultimate tension stress allowable of the material

$f_t$  = applied ultimate axial tension stress

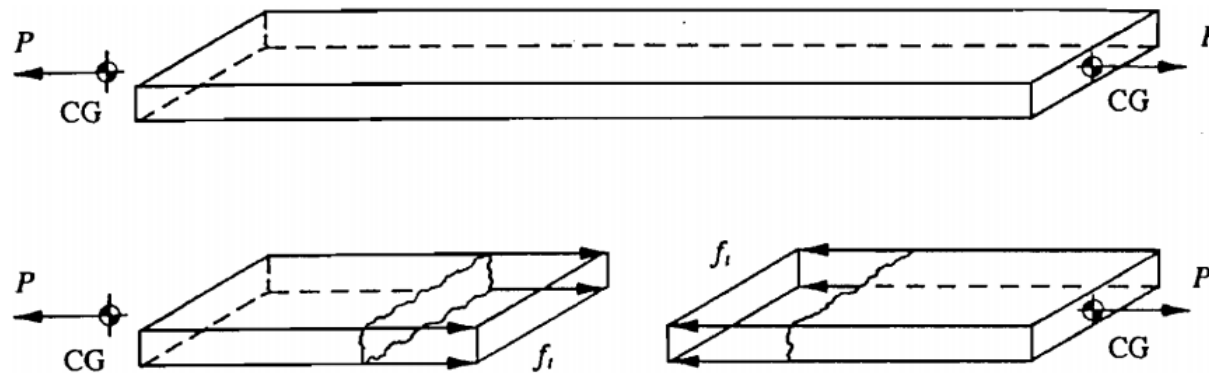
Yield Compression Failure:

$$M.S. = \frac{F_{cy}}{f_c} - 1$$

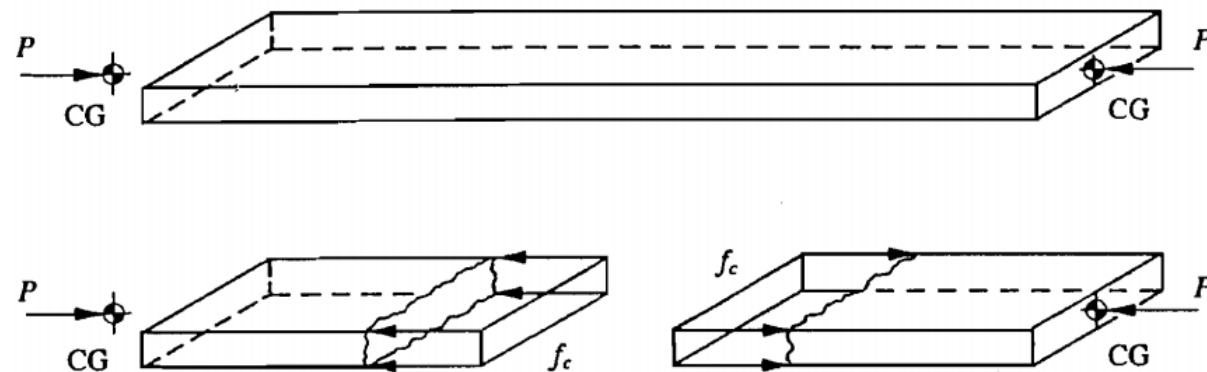
where  $F_{cy}$  = yield compression stress allowable of the material

$f_c$  = applied ultimate axial compression stress

## 2.2 Axial Compression and Tension Stresses



(a) Axial tension force.



(b) Axial compression force.

**FIGURE 2-2** (a) Internal axial tension stress distribution. (b) Internal axial compression stress distribution.

## 2.2 Axial Compression and Tension Stresses

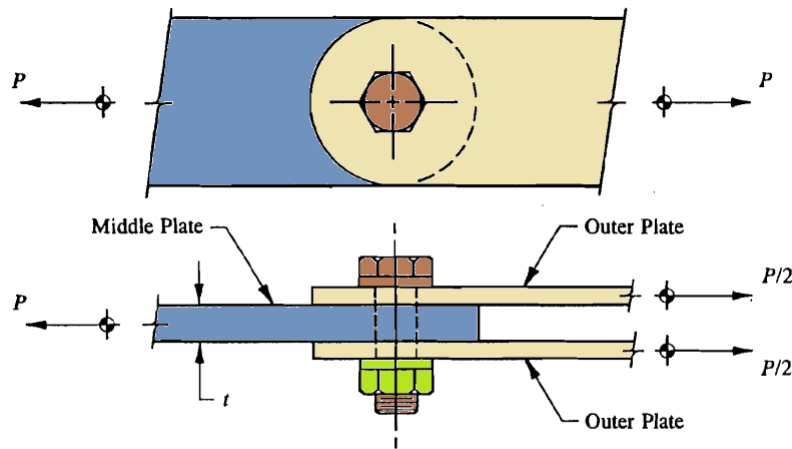


FIGURE 2-3 Double-shear lap joint loaded by a tension load  $P$ .

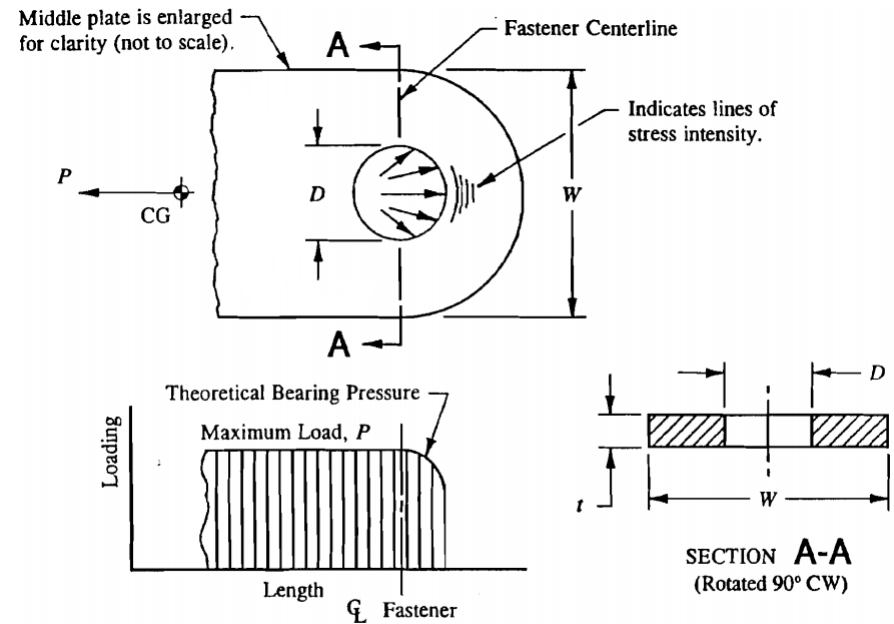


FIGURE 2-4 Axial tension diagram of the middle plate.

$$f_t = + \frac{P}{A_{net}} = + \frac{P}{(W - D) \cdot t}$$

where  $P$  = internal axial tension force

$W$  = width of the plate

$D$  = actual hole diameter

$t$  = thickness of the plate.

## 2.2 Axial Compression and Tension Stresses

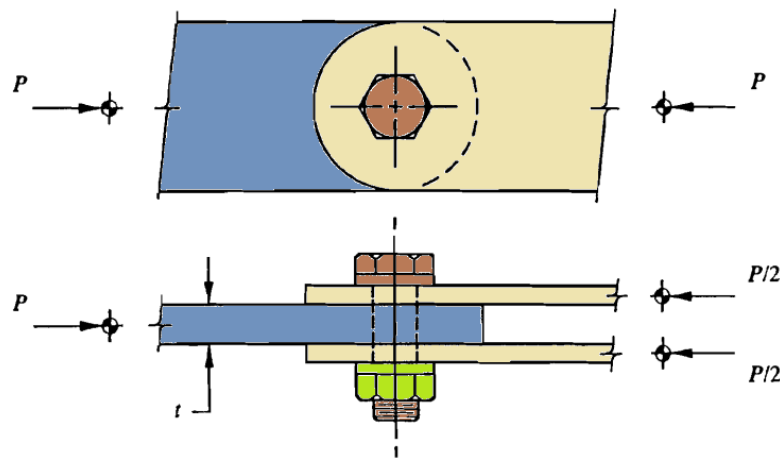


FIGURE 2-5 Double-shear lap joint loaded by a compression load  $P$ .

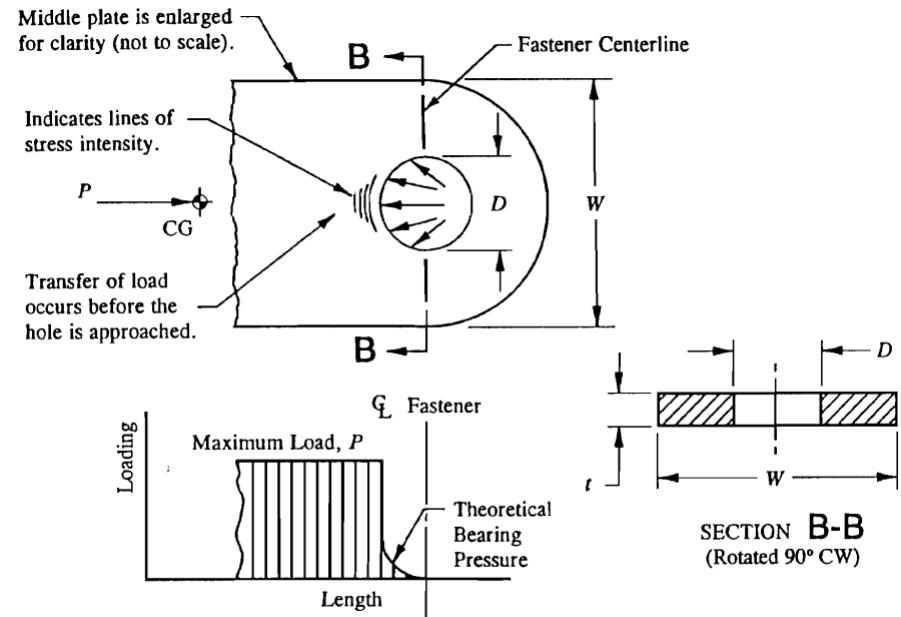


FIGURE 2-6 Axial compression diagram of the middle plate.

$$f_t = +\frac{P}{A_{gross}} = +\frac{P}{W \cdot t}$$

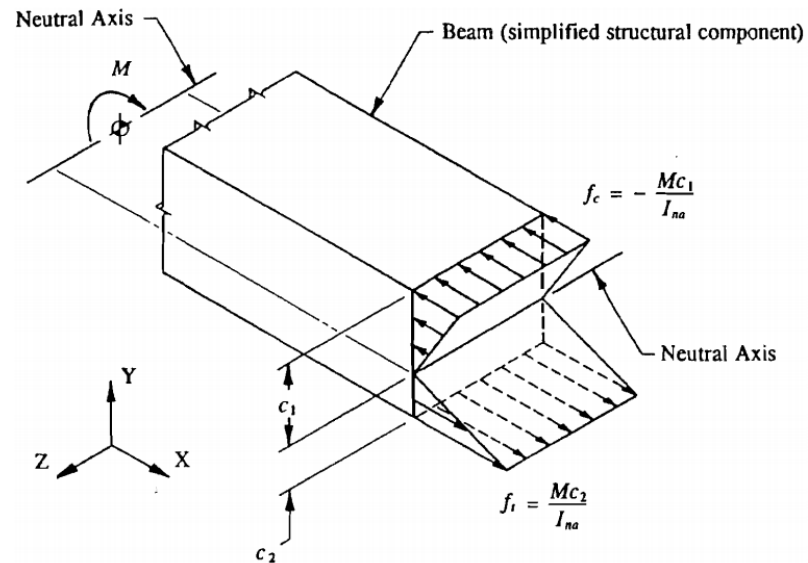
where  $P$  = internal axial compression force

$W$  = width of the plate

$t$  = thickness of the plate.



## 2.3 Bending of Beams in One Plane



$$f_b = \pm \frac{M \cdot c}{I_{na}}$$

where  $M$  = internal bending moment around the neutral axis passing through the geometric center (or centroid) of the cross-sectional area

$c$  = distance measured perpendicular from the neutral axis to any point on the cross-sectional area

$I_{na}$  = moment of inertia around the neutral axis of the entire cross-sectional area (see Appendix F for detailed calculations).

### Calculation of Moment of Inertia ( $I_{na}$ )

$$Y_{cg} = \frac{\Sigma Ay}{\Sigma A}$$

where  $\Sigma A = A_1 + A_2 + A_3 \cdots + A_n$

$$\Sigma Ay = A_1 y_1 + A_2 y_2 + A_3 y_3 + \cdots + A_n y_n$$

$A$  = area of a standard shaped section

$y$  = distance measured from the centroid (center of gravity) of a standard shaped section to an arbitrary established reference axis.

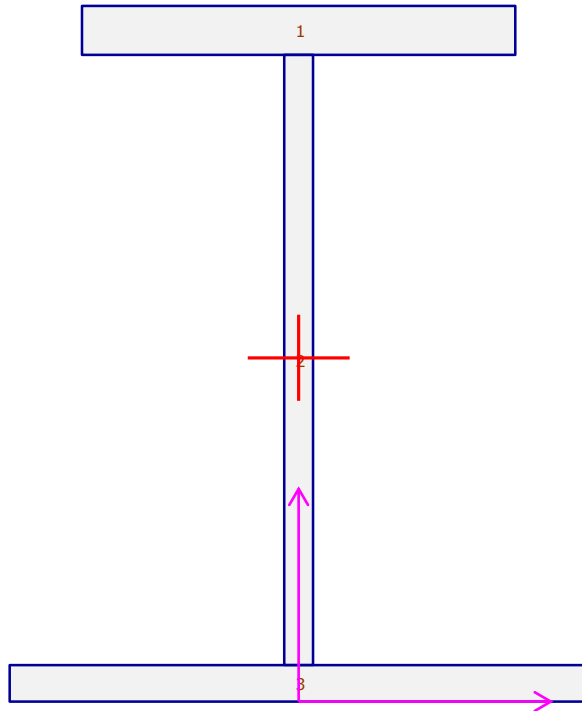
$$I_{cg} = \Sigma I_0 + \Sigma Ay^2 - Y_{cg}(\Sigma Ay)$$

where  $\Sigma I_0 = (I_0)_1 + (I_0)_2 + (I_0)_3 \cdots + (I_0)_n$

$$\Sigma Ay^2 = A_1 y_1^2 + A_2 y_2^2 + A_3 y_3^2 + \cdots + A_n y_n^2$$

$I_0$  = moment of inertia of a standard shaped section around its own centroidal axis

### Calculation of Moment of Inertia ( $I_{na}$ ) – cont'd



<i>Element#</i>	<i>b</i>	<i>h</i>	<i>y</i>	<i>A</i>	<i>Ay</i>	<i>Ay</i> <sup>2</sup>	<i>I</i> <sub>0</sub>
1	30	4	55	120	6600	363000	160
2	50	2	28	100	2800	78400	20833
3	40	3	1.5	120	180	270	90
Total				340	9580	441670	21083

$$Y_{cg} = \frac{\Sigma Ay}{\Sigma A} = \frac{9580}{340} = 28.18 \text{ mm}$$

$$\begin{aligned} I_{cg} &= \Sigma I_0 + \Sigma Ay^2 - Y_{cg}(\Sigma Ay) \\ &= 21083 + 441670 - 28.18(9580) \\ &= 192823 \text{ mm}^4 \end{aligned}$$

For a rectangular shape:

$$A = bh$$

$$I_0 = \frac{bh^3}{12}$$

## 2.3 Bending of Beams in One Plane

Ultimate Tension Failure:

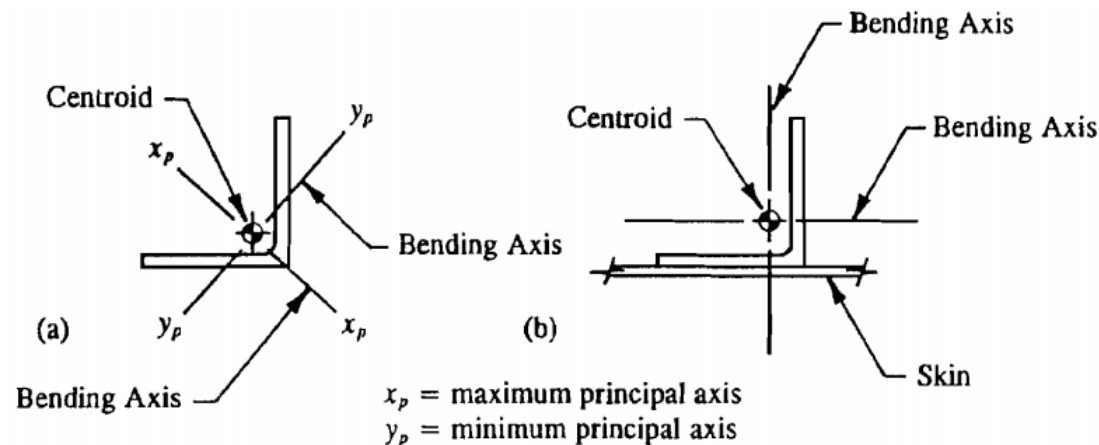
$$M.S. = \frac{F_{tu}}{f_t} - 1$$

where  $F_{tu}$  = ultimate tension stress allowable of the material  
 $f_t$  = tension ultimate bending stress

Yield Compression Failure:

$$M.S. = \frac{F_{cy}}{f_c} - 1$$

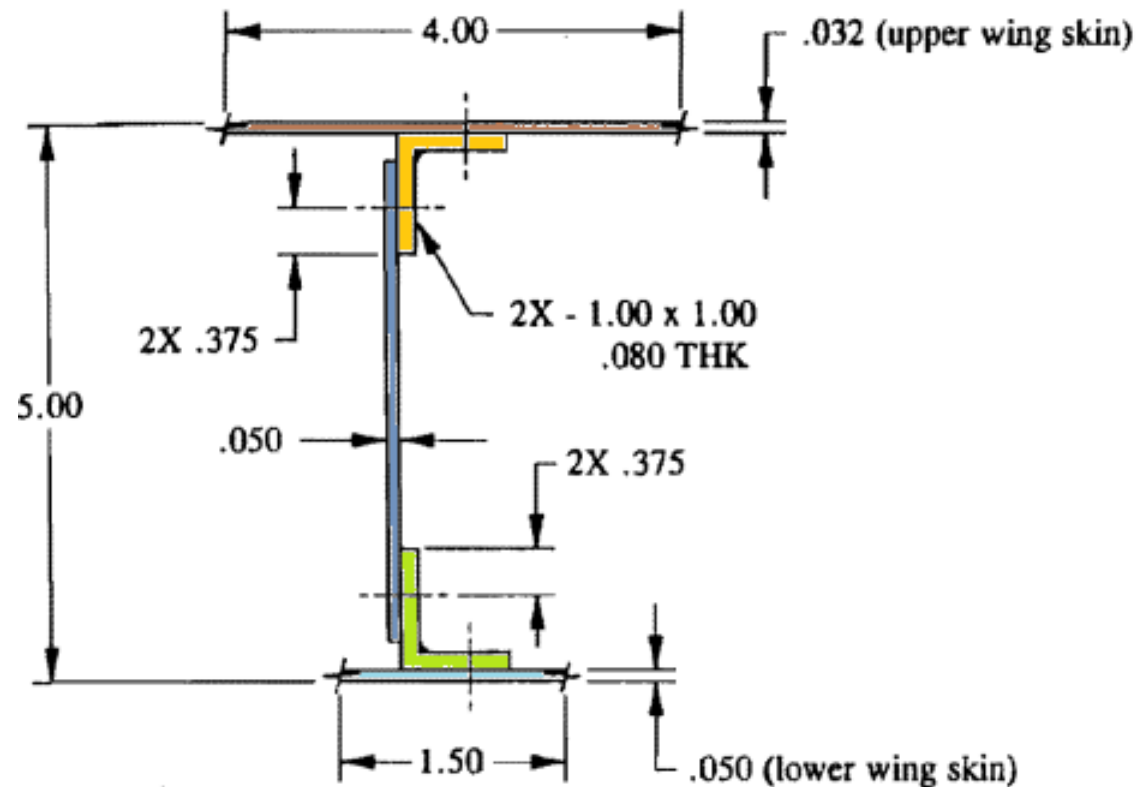
where  $F_{cy}$  = yield compression stress allowable of the material  
 $f_c$  = compression ultimate bending stress



**FIGURE 2-8** (a) Principal bending axes of a section area. (b) Skin forces bending to occur in planes either parallel or perpendicular to its surfaces.

## 2.3 Bending of Beams in One Plane

### Example 2-1:



Calculate the cross-sectional properties of Wing spar section such as  $I_{na}$ ,  $A$ ,  $Y_{cg}$ .

## 2.3 Bending of Beams in One Plane

Solution:

<i>Element#</i>	<i>b</i>	<i>h</i>	<i>y</i>	<i>A</i>	<i>Ay</i>	<i>Ay</i> <sup>2</sup>	<i>I<sub>o</sub></i>
1							
2							
3							
4							
5							
6							
7							
8							
9							
Total							